

FOREX Trading Strategies and the Efficiency Of Sterilized Intervention

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Abstract

This paper examines the role of sterilized intervention in correcting exchange rate misalignments. A model of the foreign exchange market is developed where endogenous exchange rate fluctuations develop which are unrelated to economic fundamentals, and are driven by the use of backward-looking trading strategies. It is shown how, by changing the relative profitability of available strategies, sterilized intervention can coordinate traders onto strategies based on macroeconomic fundamentals. Empirical evidence in support of the model is provided based on data from interventions by the Japanese authorities from 1991 to 2003. A key implication of the results is that intervention becomes increasingly efficient as the size of misalignment increases. These findings provide a case for a target zone for G3 exchange rates based on sterilized intervention.

1 Introduction

A persistent theme affecting international economic relations is the recurrent episodes of exchange rate 'misalignments' - periods when bilateral exchange rates do not seem to reflect relative underlying economic conditions. In the 1970's the broad shift towards floating exchange rate regimes was expected by many to facilitate rapid,

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automatic adjustment of exchange rates and, by extension economies' external positions. However in practice, the failure of the expected exchange rate stability to materialize has resulted in frequent calls for a 'target-zone' for G3 (i.e. bilateral dollar, yen and mark/euro rates) exchange rates. A key limitation of this proposal is that it requires monetary policy to commit, at least at some level, to an exchange rate target which would weaken the commitment to the maintenance of monetary stability and low inflation. In this paper, I show how these problems can be avoided by the use of sterilized intervention in operating a target zone. It is shown how such an exchange rate regime can provide increased efficiency in the foreign exchange market, and efficiently reduce the persistence of exchange rate misalignments.

Since the abandonment of the Bretton Woods system in 1973, G3 exchange rates have exhibited both high short run volatility and persistent misalignments which seem largely unconnected with macroeconomic fundamentals. Indeed, since the seminal paper by Meese and Rogoff (1983), it has been widely accepted that, at least in the short-run, models of the exchange rate based on macroeconomic fundamentals have no predictive value for floating exchange rates¹. Recent attempts to explain the large misalignments and high volatility in exchange rates have augmented macroeconomic models of exchange rates by explicitly modeling the expectation formation of currency traders, and the forecasting techniques they employ (e.g. Jeanne and Rose (2002)). There is wide spread evidence (see, e.g. Allen and Taylor (1990)) that, rather than base their forecasts solely on economic fundamentals, traders base their short-run forecasts on the past movements of exchange rates, thus extrapolating recent trends. This body of theory essentially suggests that the high volatility of exchange rates can be understood to be due to fads or bubbles generated by the use of trading rules.

Large, persistent deviations in exchange rates from equilibrium can create something of a dilemma for monetary policy makers², since monetary policy targeted at correcting exchange rate misalignments may be at odds with maintaining internal balance in the economy. In order to avoid the problems which may arise in using monetary policy to correct exchange rate misalignments, monetary authorities occasionally engage in *sterilized* foreign exchange intervention³. How effective these operations are in practice is a matter of some controversy. There is only weak evidence to suggest that intervention operations have the desired effects on exchange rates through conventional channels (i.e. by changing the 'fundamental' level of the exchange rate through portfolio balance channels, or by signaling to markets the future course of monetary policy).

¹Although more recent research (see, e.g. Taylor and Sarno (1998, 2000) has demonstrated the existence of reversion to equilibrium in the long run, this is at a lower frequency than predicted by standard macro models

²See, for example Wadhvani (2000)

³i.e. sales or purchases of foreign assets conducted by the authorities, where the effects on the domestic money base are sterilized by open market operations

An increasingly held view is that, rather than influence the fundamental level of the exchange rate, the goal of sterilized intervention operations is to limit the deviations from equilibrium due to trend-chasing by market participants. The primary contribution of this paper is to provide a theoretical model which can explain this effect consistent with observed exchange rate and intervention data, and to provide a framework for evaluating exchange rate management policies based on sterilized intervention. In particular, in July 2001 the Japanese Ministry of Finance began to publish data on all sterilized intervention operations over the last decade. This provides a valuable new source of data to gauge the success of foreign exchange intervention policy. The predictions of the model are compared with the Japanese experience with sterilized intervention in section (7) of this paper.

In order to examine the effectiveness of sterilized intervention in a framework where there can be large and persistent deviations from equilibrium, I focus on models of the forex market which explain exchange rate dynamics as in a world with heterogeneous expectations. In particular, I draw on the work of Brock and Hommes (1997, 1998, 1999) on periodically collapsing bubbles in the stock market. Traders are modelled as selecting alternative forecasting strategies on the basis of an 'evolutionary fitness rule' (e.g. realized net profits or mean squared prediction errors). A key parameter in these models is the 'intensity of choice', which measures how sensitive traders are to differences in fitness across strategies when selecting their optimal trading rule. In this framework, Brock and Hommes have demonstrated how price fluctuations are characterized by irregular switching between phases of fluctuations close to fundamentals, phases of optimism where most agents follow an upward trend, and phases of pessimism with small or large market crashes. This class of model provides an explanation for temporary speculative bubbles which are triggered by noise and reinforced by evolutionary forces.

I model the effects of sterilized foreign exchange intervention in the spirit of the portfolio-balance class of models: where intervention changes the outside supplies of domestic and foreign assets, resulting in a change in the risk premium. In general this 'portfolio balance effect' has only found weak support in empirical studies: the size of sterilized intervention operations conducted by monetary authorities are generally deemed to be too small to have significant portfolio balance effects in practice. However, in the evolutionary model presented here, changes in the supplies of assets may affect the exchange rate substantially: by inducing a change in the composition of the forecasting rules employed by traders in the market. Since realized profits are determined by the realized excess return of foreign bonds, realized profits are also influenced by the risk premium. Thus the fitness of a forecasting strategy may also be influenced by sterilized intervention operations. *The authorities may use intervention to co-ordinate traders on to trading strategies which are based on economic fundamentals rather than strategies based on technical trading rules and trend-chasing.*

The plan of the paper is as follows. Section (2) examines the role of chartism in the foreign exchange market; its role in generating exchange rate misalignments; and reviews the literature on modelling exchange rate behaviour in the presence of traders who employ chartist rules. Section (3) discusses the literature on sterilized intervention, and contrasts the approach taken in this paper with those taken previously. Section (4) presents the model, while the numerical analysis of the exchange rate dynamics in the model without intervention are examined in section (5). The effects of intervention on the exchange rate are analyzed in section (6). Section (7) provides empirical support for the model using Japanese data from 1991 to 2003. Section (8) then presents a bootstrap procedure for calculating the ex ante probability that intervention is effective, and uses the results to assess the success of Japanese intervention policy. Section (9) concludes.

2 Chartism, heterogenous expectations and exchange rate dynamics

The failure of 'structural' exchange rate models based solely on economic fundamentals is well documented (see, for example Meese and Rogoff (1983), and Frankel and Rose (1995)). The consensus view seems to be that, even allowing for overshooting effects, the volatility of exchange rates is far too high to be caused solely by volatility in the economic 'fundamental' determinants of exchange rates. In response to this failure, starting with the paper by Frankel and Froot (1986) on the dollar-mark exchange rate, there has been an increasing focus on the microstructure of foreign exchange markets in order to provide improve on established theories of exchange rate dynamics.

In the literature, particular focus has been given to the role of chartist or 'technical' rules employed by practitioners when forecasting exchange rates. While 'fundamentalist' trading strategies employed by traders are based on underlying economic determinants of the exchange rate, chartism or 'technical analysis' refers to forecasting techniques employed by traders which use past realizations of exchange rates to detect patterns which they extrapolate into the future, thus ignoring information embodied in 'structural' exchange rate models. In addition to simple charts of past exchange rate behavior, a technical analyst may use a variety of trading rules based on statistical and mathematical techniques. Popular examples include the 'head and shoulders' reversal pattern, and a variety of moving average rules for predicting turning points. The behavior of traders using chartist techniques adds a 'positive feedback' into the exchange rate, tending to accentuate deviations from the equilibrium level implied by economic fundamentals. In contrast, the behavior of traders forecasting on the basis of fundamentals adds a 'negative feedback' into the exchange rate, since when the exchange rate deviates from its equilibrium value they expect

it to return there.

There are now a large number of survey articles which document the widespread use of technical analysis in forex markets the world over. Allen and Taylor (1990) and Taylor and Allen (1992) conducted a survey of 400 chief forex dealers in the London markets, and found that approximately 90 percent of respondents based their forecasts to some degree on chartist techniques at short time horizons. As the time horizon increased from one month to one year, they found that the weight given to economic fundamentals increased. Similar studies have been carried out in the German markets (Menkhoff (1997, 1998)), the Hong Kong markets (Lui and Mole (1998)), the Hong Kong, Tokyo and Singapore markets (Cheung and Wong (1999), Cheung and Wong (2000)), and the US markets (Cheung and Chinn (1999)). In general, these studies find that the vast majority of forex dealers employ some sort of technical forecasting technique, especially at short horizons.

2.1 Chartist-Fundamentalist Models

Since the seminal paper by Frankel and Froot (1986), there have been a large number of models which attempt to model the dynamics of exchange rates with the heterogeneity of expectations which is implied by the use of fundamentalist and technical trading strategies. Most of these models can be represented in the following simple form:

$$s_t = z_t + c(E_t^{market}[s_{t+1}] - s_t) \quad (1)$$

which summarizes the 'asset market view' of the exchange rate, where s_t represents the spot exchange rate, z_t a vector of economic fundamentals. Here, $E_t^{market}[s_{t+1}]$ represents the markets expectation of the exchange rate next period, and is given by a weighted average of the forecasts made with fundamentalist and technical trading strategies, so that:

$$E_t^{market}[s_{t+1}] = \omega_t E_t^{chart}[s_{t+1}] + (1 - \omega_t) E_t^{fund}[s_{t+1}] \quad (2)$$

The various models in the literature differ principally in the process by which the weights given to each strategy, ω_t , evolve. For example, in Frankel and Froot's (1986) model, it is postulated that

$$\omega_t - \omega_{t-1} = \delta(\hat{\omega}_{t-1} - \omega_{t-1}) \quad (3)$$

where $\hat{\omega}_{t-1}$ is defined as the weight, computed *ex post*, that would have accurately

predicted the contemporaneous change in the spot rate. This weight $\hat{\omega}_{t-1}$ is defined as

$$\hat{\omega}_{t-1} = \frac{\Delta s_t}{\theta(\tilde{s} - s_{t-1})} \quad (4)$$

so that the change in weight becomes

$$\omega_t - \omega_{t-1} = \delta \frac{\Delta s_t}{\theta(\tilde{s} - s_{t-1})} - \delta \omega_{t-1} \quad (5)$$

The value of δ is crucial for the stability of the system. It determines the importance of the most recent information on updating the weights. If δ is large, then the system is stable. If it is small, then the system is unstable. Exactly how large δ needs to be for stability depends on θ .

In this model, both groups of traders behave in a different manner because they have different information sets, so that each agent is acting rationally subject to certain constraints. The information set of fundamentalists includes fundamentals, while that of chartists contains only past values of the exchange rate which they use to extrapolate into the future.

A model by De Grauwe, Dewachter, and Embrechts (1994) is similar to that of Frankel and Froot with two exceptions. First the Chartist equation takes an autoregressive form for the expected future exchange rate, so that it nests Frankel and Froot's specification. Secondly, the weights are assumed to evolve according to

$$\omega_t = 1 - \frac{1}{[1 + b(\tilde{s} - s_{t-1})^2]} \quad (6)$$

where $b > 0$. Hence the weight on the fundamentalist element is an increasing function of the deviation from economic fundamentals.

De Grauwe et al.'s rationale for this weighting function appeals to a normally distributed noise term in agents forecasts of the fundamental exchange rate. When the actual value of the exchange rate is equal to the fundamental value, half of the fundamentalists view the exchange rate as overvalued, and half as undervalued. This difference in opinion makes fundamentalist' net demand zero so they have no effect on the market. When the exchange rate is far from the true fundamental value, the majority of fundamentalists are able to agree whether the exchange rate is under or overvalued. This agreement results in the fundamentalists working together to move the exchange rate back towards fundamentals, with the degree of mean reversion increasing with the deviation from equilibrium.

2.2 The Noise Trader Approach

An alternative approach to the chartist fundamentalist framework outlined above, which builds largely on the model of the stock market by DeLong, Shleifer, Summers, and Waldman (1990), views analysts who employ chartist techniques as 'noise traders'. The essential difference between these models and those outlined above is that the presence of noise traders injects an additional element of volatility in the mean of asset returns: in contrast to fundamentalists, noise traders have imperfect knowledge of the fundamental determinants of the exchange rate. Whilst they are typically assumed to perceive the second moments of returns correctly, their perception of the first-moments is assumed to be affected by noise that is unrelated to economic fundamentals. For example, in the De Long et al (1990) model, in each period the noise traders price the asset as its true fundamental value plus an error term. Such incorrect pricing generates additional risk in holding the asset. This risk limits the willingness of the fundamentalist traders to go against the noise traders even if they know the true value of the asset. Hence arbitrageurs do not drive the asset's price toward economic fundamentals. In addition, the risk generated by the noise traders results in them earning higher average returns than those of the sophisticated traders, so that the noise traders are difficult to drive out of the market. Crucially, this is also accompanied by a higher variance of returns. In this way, the noise trader approach is well suited to modelling the excess volatility in exchange rates.

In a recent paper, Jeanne and Rose (2002) have employed the noise trader approach to provide an explanation of the excessive volatility of exchange rates under a float. In a similar spirit to the model presented here, they show that the presence of noise traders can lead to multiple equilibria in the forex market. Employing an exchange rate "target zone" following Krugman and Miller (1993), they show how an exchange rate target zone can allow the policymaker to co-ordinate activity to a low volatility equilibrium while leaving macroeconomic fundamentals unchanged. An important difference between my work and that of Jeanne and Rose (2002) is to show how essentially the same goal can be achieved by using *sterilized* intervention.

3 Transmission Mechanisms of sterilized intervention

Whilst a complete survey of the literature on intervention is beyond the scope of this paper, it is important to describe the difference between the approach taken here and those of previous studies. In particular, the majority of previous literature has focused on the workings of sterilized intervention either through the portfolio balance effect, or via the signaling of future government policies. The portfolio-balance

effect can be motivated by the theory of mean-variance portfolio selection, where risk averse agents choose their optimal portfolio, which is composed of domestic and foreign currencies and bonds, so as to maximize their terminal wealth. Domestic and foreign bonds are assumed to be imperfect substitutes and Ricardian equivalence is absent. Then, sterilized intervention will affect the exchange rate when agents readjust their portfolios of domestic and foreign-denominated bonds in line with the changes in the outside supplies of those assets. This approach has been pursued by Dominguez and Frankel (1993a,b), with critiques provided by Humpage (1988), Obstfeld (1989), and Ghosh (1992). Given the small size of intervention operations compared with the size of asset stocks, there can be little surprise that evidence in support of the theory is in short supply.

The second channel whereby intervention can influence exchange rates is termed the signalling, or information channel. This view of intervention as a signal of the central banks's future monetary policy implies that a sterilized purchase of foreign currency is expected to lead to a depreciation of the exchange rate if the purchase is assumed to signal a more expansionary domestic monetary policy. Klien and Rosengen (1991) find no consistent relationship between intervention and monetary policy, while Kaminsky and Lewis (1996) report that the impact of intervention on the exchange rate has sometimes been inconsistent with the implied monetary policy. Humpage (1997) concludes that the US authorities in the 1990's had no information superior to the market so that intervention could not be viewed as signalling new information about monetary policy. The apparent failure of the signalling hypothesis is perhaps not so surprising given the monetary model's failure to predict exchange rate movements. If there is little direct transmission between changes in relative money supplies and movements in short term exchange rates, then it is also unclear why signalling monetary policy should have a substantial effect on exchange rates.

The general view taken by monetary authorities of how sterilized intervention effects exchange rates in practice seems to differ markedly from the 'traditional' channels outlined above. Rather than attempt to influence the *equilibrium* level of the exchange rate, sterilized intervention operations are generally undertaken when the authorities view the exchange rate to be under or overvalued. Thus interventions are carried out with the intent of reducing current *misalignments*, or indeed excessive volatility. The rationale for this kind of intervention operation is best viewed in the light of the use of technical analysis by FOREX traders discussed in the previous section. Wadhvani (2000) notes "Under some circumstances, FX intervention can give the fundamentals-based traders greater confidence to initiate positions during overshoots. Alternatively, in an over-extended market, intervention can sometimes directly affect the behaviour of the momentum-based traders." The analogy employed by Dominguez and Frankel (1993b) for FX intervention emphasizes this nicely. They liken the role of intervention to the role of herd dogs amongst cattle. Clearly a small number of dogs cannot always sustain control of the steers. So, when a stampede gets underway because each panicked steer is following its neighbours,

the herd can wander off quite far from their initially desired direction. However, the dogs can be helpful in a stampede because, by turning a few steers around, they might induce the herd to follow.

While this provides a strong intuitive description of how intervention may 'prick bubbles' in exchange rates, to my knowledge, there has as yet been no attempt to rigorously examine the mechanisms by which intervention may produce this kind of effect. There are, however, two empirical studies which test for the kinds of effect modelled in this paper. Hung (1997), employs a noise trading framework to explain the impact of US intervention on exchange rate volatility. He finds that intervention by the US FED reduced both yen/dollar and DM/dollar exchange rate volatilities during 1985-1986, but increased them during 1987-1989. Taylor (2003) examines the effectiveness of intervention within the context of a Markov-switching model for the real exchange rate. The probability of switching between stable and unstable regimes depends nonlinearly upon intervention, the degree of misalignment and the duration of the regime. Applying this to dollar-mark data for the period 1985-1998, he finds that intervention increases the probability of stability when the rate is misaligned, and that its influence grows with the degree of misalignment.

4 Sterilized Intervention in a Model with Trader Heterogeneity

4.1 Evolutionary Dynamics

While the chartist-fundamentalist models outlined in section (2.1) have considerable intuitive appeal there seems to be room for improvement on a number of fronts. The key implication of Frankel and Froot's model is that in the short run, the weight of traders who employ each strategy changes according to the group's respective wealths. As long as chartists continue to make money, in the long run fundamentalists will be driven out of the market entirely. This would imply an explosive path for the exchange rate, with the potential for no mean reversion. An appealing alternative to this approach is developed in a series of papers by Brock and Hommes (1997, 1998, 1999). I draw on their work to model how beliefs are updated over time i.e. how the fractions ω_{ht} evolve. The framework they develop represents a significant advance on previous studies for a number of reasons. Firstly, rather than select the appropriate trading strategy on the basis of the essentially ad hoc measures proposed in other studies, agents select forecast rules in a (boundedly) rational manner, on the basis of a 'fitness' or 'performance' measure which is a function of the profits from each strategy. Agents are boundedly rational in the sense that they choose the strategy with the highest fitness, so that strategies with highest performance will dominate the evolutionary dynamics. Modeling of the selection of trading strategies

draws on random utility models and the theory of discrete choice.

In order to keep the discussion as simple as possible, heterogeneity in the expectations of traders is modeled in the following way. Each period, given the level of last periods exchange rate s_t , traders may either pay a fixed cost⁴, C , to purchase a forecast based on macroeconomic fundamentals:

$$f_{fund,t} = v s_{t-1} \quad 0 < v < 1. \quad (7)$$

or they may simply extrapolate recent trends by employing a generic chartist rule:

$$f_{chart,t} = g s_{t-1} \quad g > 1, \quad (8)$$

The assumption that $g > 1$ implies that traders following the chartist strategy expect the overvaluation of a currency to extend further next period. $0 < v < 1$ implies that traders using the fundamentalist strategy expect any currency overvaluation to be reduced next period.

The model of strategy choice extends work by Brock and Hommes (1997), so that traders select their strategy on basis of an evolutionary 'fitness measure'. There are a number of possible candidates for the variables which influence the fitness of each strategy. Brock and Hommes specify the fitness rule as depending on accumulated realized profits, while Gaunersdorfer and Hommes (2000), specify evolutionary fitness given by risk adjusted profits⁵. In Brock and Hommes (1998) it is shown how, with realized profits as the fitness measure, stock prices may be characterized by endogenously expanding and collapsing bubbles. However, with this simple form this result is only obtained either when traders beliefs are biased in some way, or when traders employing *all* strategies continuously make losses. One method of rectifying this situation to adopt the approach of Gaunersdorfer and Hommes (2000), for example. In that paper, the fitness of any strategy is then assumed to be conditioned upon the deviation from equilibrium. This has the effect of ruling out explosive paths

⁴This cost represents the information gathering cost necessary to correctly forecast the level of the equilibrium exchange rate, and may by no means be insignificant. For example, De Grauwe, Dewachter, and Embrechts (1994) note that in a world where authorities are bound by few commitments the number of possible future paths of debt and money is increased. This is due, both to the move toward floating exchange rates, and also to the dramatic increase in international capital mobility allowing for a much wider range of debt financing options than before. The net effect is that correct forecasting of the equilibrium exchange rate is more difficult, and hence costly.

⁵In a recent paper, De Grauwe and Grimaldi (2004) develop a model of the forex market where the fitness measure is a function of the one-period earnings of investing in the foreign asset. The authors point out that this ensures strategy selection is influenced only by the relative *profitability* of the strategies, and not by the amount invested. They motivate this specification of the fitness rule by it's emphasis on the selection of strategy independent of agent's stock of wealth

for the asset price, even when the profits to traders following chartist strategies increase as more traders co-ordinate onto that strategy. Hommes (2000) justifies this conditioning upon fundamentals as a weakening of the transversality (no bubbles) condition in a perfectly rational world, allowing for temporary speculative bubbles.

In this paper the fitness of each available strategy is assumed to be given by a weighted function of past realized profits, and expected profits next period. Strategy choice therefore entails both forward and backward looking behavior. The fitness of strategy h then takes the form:

$$U_{h,t} = (1 - \delta)\rho_t z_{h,t-1} + \delta E_{h,t}[\rho_{t+1} z_{h,t}] - C_h, \quad (9)$$

Where ρ_t is the excess return on the risky asset, $z_{h,t}$ represents the demand for the risky asset by traders employing strategy h , and δ is an exogenous parameter, giving the weight traders place on the expected component of the fitness function. C_h represents the cost of purchasing each strategy, and for the chartist strategy is assumed to be zero so that $C_c = 0$, and $C_f = c$. This form of fitness function assumes that, when deciding upon which strategy to employ, traders compare both the realized profits last period, and the size of expected profits next period from each strategy.

While the fitness measures of all trading strategies are publicly available, they are subject to noise arising from shifts in preferences. The random utility model for evaluating such rules is given by

$$\tilde{U}_{ht} = U_{ht} + \varepsilon_{ht} \quad (10)$$

where ε_{ht} is IID across $h = 1, \dots, H$ and drawn from a double exponential distribution. As the number of traders goes to infinity, the probability that an agent chooses the fundamentalist strategy is given by the discrete choice model:

$$\omega_{f,t} = \frac{1}{1 + \exp(\beta(U_{c,t-1} - U_{f,t-1}))}. \quad (11)$$

Thus the higher the fitness of trading strategy h , the more traders will select h . The 'intensity of choice' parameter β measures the proportion of traders who select the optimal prediction strategy. β is inversely related to the variance of the noise terms ε_{ht} . If $\beta = 0$, the variance of the noise is infinite, so that traders are unable to discern any difference in the fitnesses of available strategies and all fractions (11) will be fixed over time to equal $1/H$. The other extreme case $\beta = \infty$ corresponds to the case without noise, so that the deterministic part of the fitness measure can be observed perfectly, and in each period, *all* traders choose the optimal forecast. In

general, higher values of the intensity of choice β represent an increase in the degree of rationality of traders when selecting their strategy.

To summarize: financial markets are modeled as an evolutionary system between competing trading strategies, following Brock and Hommes (1997,1998). While all traders are aware of the fitness of each competing strategy, they are unaware of the proportion of traders employing each strategy at any given time. To compensate for this, all traders attach a small exogenous probability to a sudden reversion in asset prices to fundamental, and take potential gains or losses arising from such a reversion into account when evaluating alternative strategies.

As will be seen later, these evolutionary dynamics can result in the exchange rate displaying persistent, endogenous, exchange rate fluctuations which are triggered by a rational choice between speculative, self-fulfilling trading strategies.

4.2 Foreign Exchange Market

We begin by considering a simple monetary model of the exchange rate. With money market equilibrium in the home and foreign countries, and assuming PPP holds continuously

$$m_t - p_t = \gamma y_t - \alpha i_t \quad (12)$$

$$m_t^* - p_t^* = \gamma y_t^* - \alpha i_t^* \quad (13)$$

$$s_t = p_t - p_t^* \quad (14)$$

so that:

$$s_t = (m_t - m_t^*) - \gamma(y_t - y_t^*) + \alpha(i_t - i_t^*) \quad (15)$$

Sterilized foreign exchange intervention is introduced into the model via the portfolio-balance view that changes in the outside supplies of assets denominated in domestic and foreign currency alter the risk premium on foreign bonds. Let $E_{h,t}$ denote the forecast of a trader employing strategy type h . Then a trader employing forecasting strategy h who has entered the foreign bonds market invests $z_{h,t}$ in foreign bonds so as to maximize the utility of her end of period wealth, expressed in terms of the domestic currency. Then, traders following strategy h are assumed to have the following portfolio allocation problem at time t :

$$\max_{z_{h,t}} U_{h,t} = E_{h,t}[-\exp(-aW_{h,t+1})] \quad (16)$$

where $W_{h,t+1}$ is the end of period wealth of trader type h . Wealth dynamics are given by

$$W_{h,t+1} = \alpha(1+i)w_{h,t} + (1-\alpha)(\rho_{t+1})z_{h,t} \quad (17)$$

where ρ_{t+1} is the excess return on foreign bonds between t and $t+1$, and α is the share of the investors portfolio in domestic bonds. It is well known that maximizing (16) when returns are normally distributed, is equivalent to the problem

$$\max_{z_{h,t}} \{E_{h,t}[W_{t+1}] - \frac{a}{2}V_{h,t}[W_{t+1}]\} \quad (18)$$

so that, solving the maximization problem we have

$$z_{h,t} = \frac{E_{h,t}[\rho_{t+1}]}{aV_{h,t}[\rho_{t+1}]} \quad (19)$$

Assuming that $V_{h,t}[\rho_{t+1}] = \sigma^2 \forall h$, the demand for foreign bonds by a trader of type h can be written as

$$z_{h,t} = \frac{E_{h,t}[\rho_{t+1}]}{a\sigma^2} \quad (20)$$

so that total demand for foreign bonds is given by

$$Z_t = \sum_{h=1}^H \omega_{h,t} \frac{E_{h,t}[\rho_{t+1}]}{a\sigma^2} \quad (21)$$

In equilibrium, equating world demand for foreign bonds with world supply, it can be shown (see, for example Flood and Marion (2000)) that the linearized risk premium on foreign bonds is given by

$$a\sigma^2(c + b_t - b_t^* - s_t) = i_t - i_t^* - \sum_{h=1}^H \omega_{h,t} E_{h,t}[s_{t+1} - s_t] \quad (22)$$

Equation (22) is the interest parity condition, which shows how the domestic interest rate deviates from the foreign interest rate, i_t^* , by the markets' expectation of the rates of change of the exchange rate, $\sum_{h=1}^H E_{h,t}[s_{t+1} - s_t]$, plus a time varying risk premium, $a\sigma^2(\cdot)$. The risk premium is influenced by the relative private holdings

of domestic and foreign government securities, agents's attitudes towards risk, and uncertainty about the future exchange rate. The term $(b_t - b_t^* - s_t)$ measures the worldwide private holdings of domestic relative to foreign government securities. The term $a\sigma^2(\cdot)$ summarizes how desired asset holdings are influenced by tastes toward risk and uncertainty about returns. If intervention operations increase the supply of domestic relative to foreign assets held by the market, then investors will require a higher expected return on domestic assets to willingly hold the larger outstanding stock, leading to a depreciation of the domestic currency.

In order to directly consider the effects of sterilized intervention on the determination of the exchange rate, note that the relative stock supplies of outside assets are equivalent to cumulated intervention⁶ Υ_t .

So that (22) may be written

$$s_t = (i_t^* - i_t) + a\sigma^2\Upsilon_t + \sum_{h=1}^H \omega_{h,t} E_{h,t}[s_{t+1}] \quad (26)$$

Integrating the interest parity relation (26) and the simple monetary model implied by (15) we can express the market equilibrium equation as

$$s_t = \frac{1}{(1 + \alpha)} (m_t - m_t^* - \gamma(y_t - y_t^*)) + \frac{\alpha}{(1 + \alpha)} a\sigma^2\Upsilon_t + \frac{\alpha}{(1 + \alpha)} \sum_{h=1}^H \omega_{h,t} E_{h,t}[s_{t+1}] \quad (27)$$

4.3 Updating Beliefs

In the above model of the forex market, realized profits may be written as

$$\pi_{h,t} = (i_t^* - i_t + (s_{t+1} - s_t)) \frac{(i_t^* - i_t + E_{h,t}[s_{t+1} - s_t])}{a\sigma^2} \quad (28)$$

Now, denoting relative money velocity $z_t = (m_t - m_t^*) - \gamma(y_t - y_t^*)$, and substituting in for the interest differential from (15), the excess return at time t from holding foreign bonds is given by the following expression

⁶Sterilized intervention is equivalent to the *change* in demand for foreign bonds, i.e.

$$a\sigma^2\Upsilon_t = E_{h,t}[\rho_{t+1}] - E_{h,t-1}[\rho_t] \quad (23)$$

$$= E_{h,t}[\rho_{t+1}](1 - \mathcal{L}) \quad (24)$$

So that

$$E_{h,t}[\rho_{t+1}] = (1 - \mathcal{L})^{-1} a\sigma^2\Upsilon_t \quad (25)$$

$$\rho_t = (i_{t-1}^* - i_{t-1}) + (s_t - s_{t-1}) \quad (29)$$

$$= \frac{1}{\alpha} z_{t-1} + s_t - \frac{(1+\alpha)}{\alpha} s_{t-1} \quad (30)$$

It is useful to rewrite the model in terms of deviations from the benchmark fundamental, rational expectations (perfect foresight) level of the exchange rate, s_t^* . So that

$$s_t = x_t + s_t^* \quad (31)$$

where x_t is the deviation of the exchange rate from fundamentals. Then,

$$\rho_t = x_t + s_t^* + \frac{1}{\alpha} z_{t-1} - \frac{(1+\alpha)}{\alpha} (x_{t-1} + s_{t-1}^*) \quad (32)$$

$$= x_t - \frac{(1+\alpha)}{\alpha} x_{t-1} + s_t^* + \frac{1}{\alpha} z_{t-1} - \frac{(1+\alpha)}{\alpha} s_{t-1}^* \quad (33)$$

Since from the equilibrium equation (27)

$$s_t^* + \frac{1}{\alpha} z_{t-1} - \frac{(1+\alpha)}{\alpha} s_{t-1}^* = -a\sigma^2 \Upsilon_{t-1} \quad (34)$$

we then have

$$\rho_t = x_t - \frac{(1+\alpha)}{\alpha} x_{t-1} - a\sigma^2 \Upsilon_{t-1} \quad (35)$$

Using this to express the fitness rule (9) for each strategy in terms of deviations from fundamental equilibrium, the relative fitness of the chartist strategy can be simplified considerably. Letting $q = g - v$, and combining the model of equilibrium in the forex market with the equations governing updating of beliefs, the general form of the model may then be written as

$$x_t = \frac{\alpha g}{(1 + \alpha)} x_{t-1} - \frac{\alpha q}{(1 + \alpha)} \omega_{f,t-1} x_{t-1} + \varepsilon_t \quad (36)$$

$$\omega_{f,t} = \frac{1}{1 + \exp(\beta(U_{c,t-1} - U_{f,t-1}))} \quad (37)$$

$$U_{c,t-1} - U_{f,t-1} = \left((1 - \delta)x_{t-1} - \frac{(1 + \delta)(1 + \alpha)}{\alpha} x_{t-2} + \delta(g + v)x_{t-3} \right. \quad (38)$$

$$\left. - (1 + \delta)a\sigma^2 \Upsilon_{t-2} \right) \frac{qx_{t-3}}{a\sigma^2} + c. \quad (39)$$

Note that a stochastic term, ε_t , is introduced to represent model approximation error

5 Exchange Rate Dynamics in the Absence of Intervention

The evolutionary selection of trading strategies by market participants described in previous sections can lead to endogenous fluctuations in the exchange rate which are unrelated to economic fundamentals. In this section, the processes by which these fluctuations arise are examined in detail. Initially, exchange rate dynamics are examined in the case where there is no intervention, i.e. where the outside supply of foreign bonds is normalized to zero. In conducting the stability analysis, we will initially consider the dynamics of the deterministic skeleton of (36) - (39). However, later we will conduct some simulation exercises in the presence of iid noise to better gauge the effect of interventions.

5.1 Analysis of Steady States

In the absence of intervention, the model closely resembles that considered by Brock and Hommes (1998). A key parameter governing the dynamics of the system (36) - (39) is the intensity of choice β . Recall that this parameter captures how sensitive the mass of traders are to deviations in expected profits from the available strategies. Thus, as β increases, the proportion of traders on the most profitable strategy in net terms increases. The existence and stability of the steady states of the system as the intensity of choice varies is examined in the following lemma.

Lemma 1 (Existence and stability of steady states with $\Upsilon = 0$). *Let the steady state mass of chartists, $w^* = \frac{g}{q} - \frac{1+\alpha}{q\alpha}$. Then for $g < \frac{1+\alpha}{\alpha}$, there is a unique fundamental steady state $E_0 = (0, \omega^*)$. For $g > \frac{1+\alpha}{\alpha}$, there are two possibilities:*

1. E_0 is the unique steady state if either

$$(a) \quad (g + v) > 2 + \left(\frac{1 + \delta}{\alpha\delta}\right) \quad \text{and} \quad \exp(\beta C) > \frac{1 - \omega^*}{\omega^*}$$

or

$$(b) \quad (g + v) < 2 + \left(\frac{1 + \delta}{\alpha\delta}\right) \quad \text{and} \quad \exp(\beta C) < \frac{1 - \omega^*}{\omega^*}$$

2. The fundamental steady state E_0 is unstable, and there are two stable, non-fundamental steady states $E_1 = (x^*, \omega^*)$, $E_2 = (-x^*, \omega^*)$ if either:

$$(a) \quad (g + v) < 2 + \left(\frac{1 + \delta}{\alpha\delta}\right) \quad \text{and} \quad \exp(\beta C) > \frac{1 - \omega^*}{\omega^*}$$

or

$$(b) \quad (g + v) > 2 + \left(\frac{1 + \delta}{\alpha\delta}\right) \quad \text{and} \quad \exp(\beta C) < \frac{1 - \omega^*}{\omega^*}$$

From Lemma (1), we can see how the number and stability of the steady states changes according to the degree of extrapolation by chartists g . When this is low, ($0 < g < \frac{(1+\alpha)}{\alpha}$), then the fundamental steady state E_0 is globally stable. If costs $C = 0$, half of the traders are trend chasers, and half are fundamentalists for any value of β . If $c > 0$, then the mass of traders employing the fundamentalist strategy decreases to zero as β or c tends to $+\infty$. As the intensity of choice β increases the proportion of traders on the most profitable strategy in net terms increases.

When chartists strongly extrapolate recent trends, so that $g > \frac{(1+\alpha)}{\alpha}$ and there are positive information costs for obtaining the fundamentalist strategy, for large enough value of β , the qualitative behavior of the system changes. For $\beta = 0$, $\exp(\beta c) < \frac{\omega^*}{(1-\omega^*)}$, whereas for large β , $\exp(\beta c) > \frac{\omega^*}{(1-\omega^*)}$. Hence, as the intensity of choice increases, a *pitchfork* bifurcation occurs for some $\beta = \beta^*$, in which the fundamental steady state E_1 becomes unstable and two additional stable, non-fundamental steady states $E_2 = (x^*, \omega^*)$ and $E_3 = (-x^*, \omega^*)$ are created, one above and one below the fundamental. These non-fundamental steady states correspond to a special case where there are a constant proportion of traders using both the fundamentalist and chartist strategies. However, if these steady states were stable, it would imply that traders do not adjust their strategy choice despite the fact that both strategies consistently produce incorrect forecasts. However, as we shall see in the next section, as the value of β increases further, the non-fundamental steady states become unstable leading to endogenous fluctuations in the exchange rate. It

is these fluctuations around the non-fundamental steady states which we refer to as 'bubbles' in this paper.

5.2 Endogenous Exchange Rate Fluctuations

As β increases still further above β^* , the exchange rate begins to exhibit periodic fluctuations about the non-fundamental steady states. This process is described in Lemma (2).

Lemma 2 (Secondary bifurcation, $\theta = 0$). *Let E_1 and E_2 be the non-fundamental steady states as in Lemma (1). Assume $c > 0$ and $g > \frac{(1+\alpha)}{\alpha}$, and let β^* be the pitch-fork bifurcation value. Then as β increases above β^* there exists some β^{**} at which E_1 and E_2 undergo a Hopf bifurcation at which E_1 and E_2 become unstable, and two limit cycles are created about E_1 and E_2 .*

For sufficiently high values of β the exchange rate dynamics are characterized by limit cycles around the unstable, non-fundamental steady states. An example is illustrated in the phase diagram, figure (1) below.

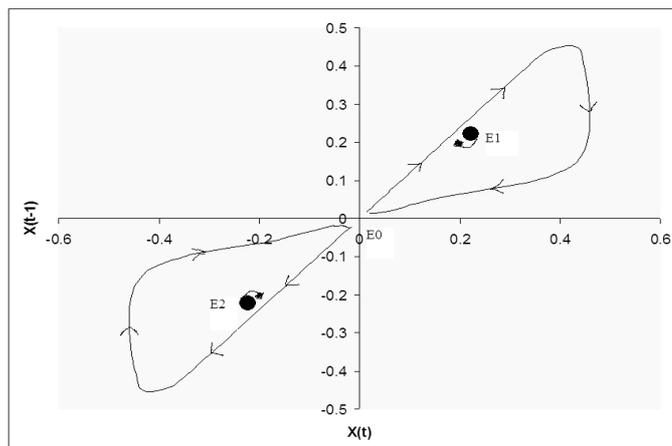


Figure 1: Limit cycles around the unstable non-fundamental steady states E_1 and E_2 .

It is useful at this point to examine in some detail the processes at work in the market which lead to these periodic fluctuations, as illustrated in figure (2). The top two figures (a) and (b) show the path of the deviation of the exchange rate from equilibrium. It is characterized by movements away from equilibrium which gain in momentum, before a short dip in towards the fundamental, followed by a market crash. The one period excess returns to holding foreign bonds are illustrated in figures (c) and (d). Returns grow to a maximum as the exchange rate moves away

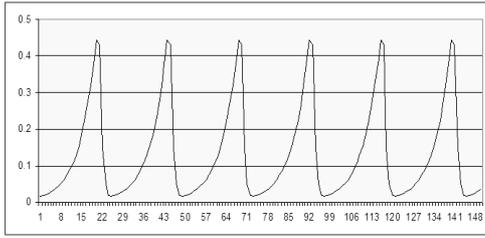
from equilibrium to its peak, and become negative when the crash occurs. Figures (e) and (f) show the demands for foreign bonds by traders employing the chartist and fundamentalist strategies respectively. It is clear from the figures that chartists purchase larger quantities of foreign bonds the further the exchange rate gets from equilibrium, while fundamentalists sell foreign bonds ever more strongly.

Profits to traders following each type of strategy are shown in figures (g) and (h). Clearly, as long as the exchange rate moves away from the fundamental, chartists make profits, however, when the market crashes large losses are suffered by those traders who continue to follow the chartist strategy. Fundamentalists endure losses from the cost of obtaining the strategy, together with trading losses that rise as the deviation from equilibrium increases. However, once the market reversal begins, large gains are made.

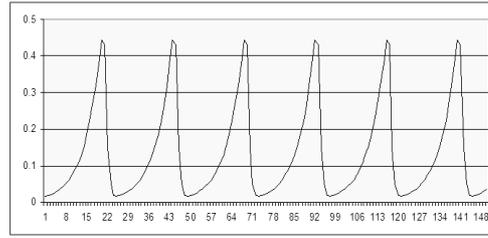
Finally, the proportions of traders following each strategy are illustrated in figures (i) and (j). Crucially, as the deviation from equilibrium rises, the mass of traders on the chartist strategy falls. This is because the expected losses in the event of a crash increase the further the deviation from equilibrium, making the chartist strategy increasingly unattractive. In a similar manner, the mass of traders employing the fundamentalist strategy rises with the deviation from fundamentals because the expected gains in the event of a sudden reversal are increasing. The processes by which the cycle of growth and collapse of bubbles now becomes clear. Due to costs of obtaining the fundamental strategy, the majority of traders employ a chartist strategy when the exchange rate is close to fundamentals. The chartists push the exchange rate further and further away from equilibrium in the desire to reap profits from the (self-fulfilling) bubble. However, as the larger the bubble becomes, the greater the potential for losses in the event of a sudden collapse, this risk leads traders to switch to the fundamentalist strategy at an increasing rate. As soon as the mass of traders on the chartist strategy is no longer sufficient to maintain the growth of the bubble, the exchange rate dips slightly toward the fundamental, profits to the chartist strategy become negative prompting a sudden switch to the fundamentalist strategy, and the bubble collapses.

Given this apparently simple, periodic behavior in the absence of noise, it is tempting to question whether such apparently predictable fluctuations could represent a plausible description of exchange rate behavior. One may postulate that given apparently predictable turning points of fluctuations, traders would soon learn to exploit the opportunities for profits that they represent, such that these kinds of fluctuations could be ruled out by a process of backward induction. However, once the dynamics of the exchange rate are examined in the presence of noise, it immediately becomes clear that predicting market crashes or turning points is no easy task.

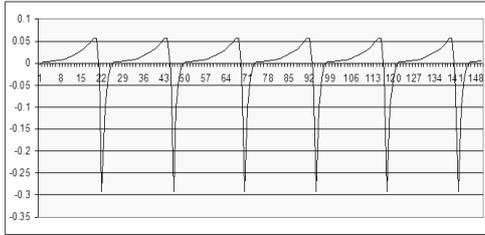
The same model is illustrated in figures (3) (a) and (b) in the case where the system is being buffeted with noise. Here, the random shocks to the system result in flips between the steady states E_1 and E_2 , and periods close to the fundamental.



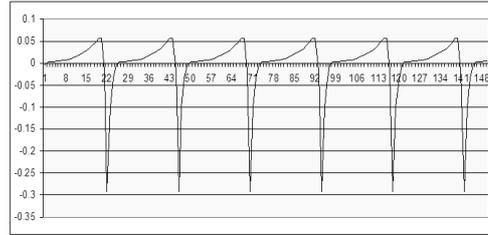
(a) Deviations from equilibrium.



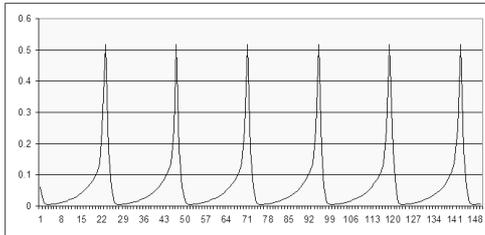
(b) Deviations from equilibrium.



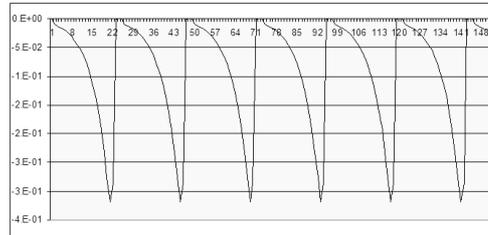
(c) Excess returns on foreign bonds



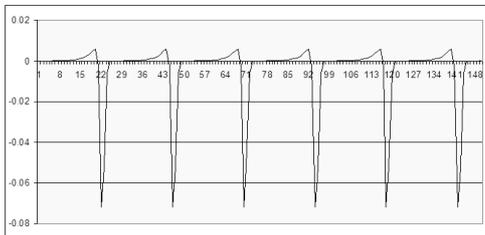
(d) Excess returns on foreign bonds



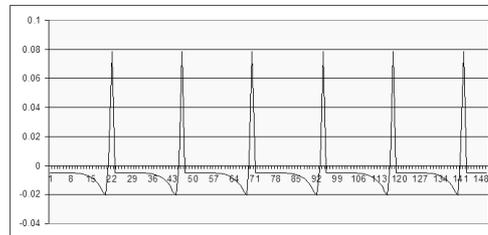
(e) Demand by chartists



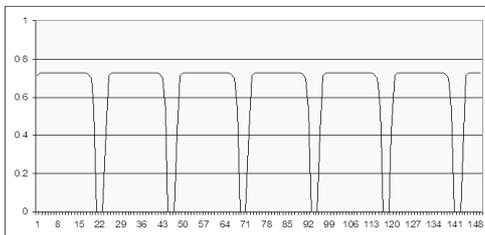
(f) Demand by fundamentalists



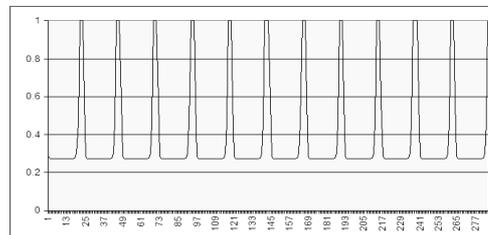
(g) Profits to chartists



(h) Profits to fundamentalists



(i) Weight on chartist strategy



(j) Weight on fundamentalist strategy

Figure 2: Periodically collapsing bubbles in the forex market.

Thus, in the absence of intervention, the exchange rate can exhibit prolonged deviations from equilibrium, periods of high volatility, and market crashes. Qualitatively, this behavior is consistent with the observed behavior of floating exchange rates. Furthermore, it is clear that the shocks to the exchange rate result in fluctuations which appear random to the naked eye, thereby making the exploitation of periodic fluctuations by arbitrageurs all but impossible.

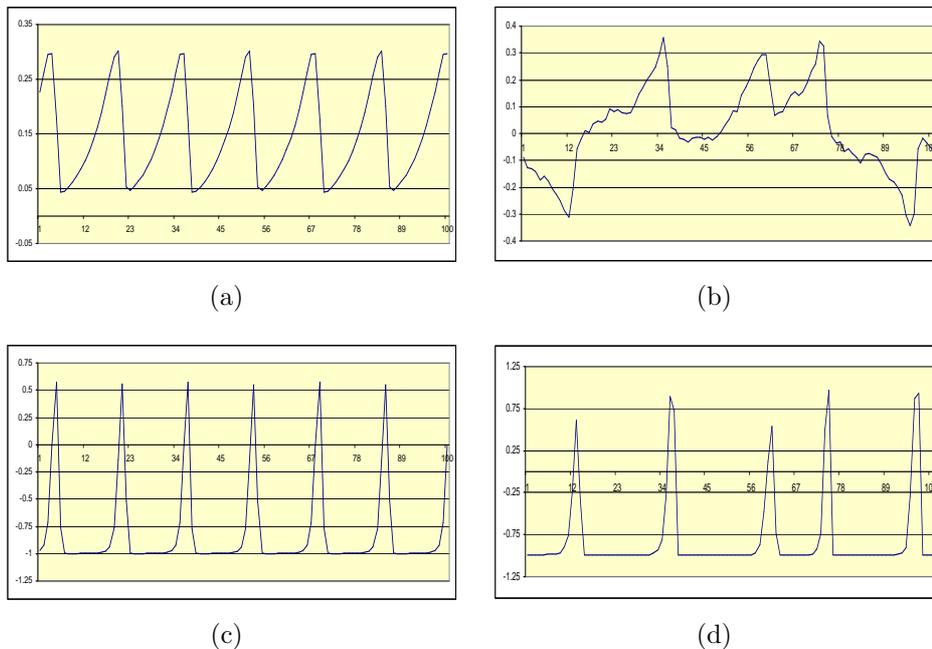


Figure 3: Hopf bifurcations without noise (a) and with noise (b). With corresponding ratios of fundamentalists to chartists, (c) and (d) .

6 Official Intervention Policy

6.1 Exchange Rate Dynamics Under Intervention

The previous section examined the processes which can lead to endogenous exchange rate fluctuations and persistent deviations from levels implied by economic fundamentals. From the monetary authorities point of view a key question is to what extent can intervention reduce these prolonged deviations from equilibrium, i.e. 'burst' bubbles. In this section we begin by examining the effects of a one-off intervention operation on the exchange rate dynamics. We then proceed to discuss how the authorities can exploit the processes governing the endogenous collapse of exchange rate bubbles to increase the efficiency of intervention operations.

Lemma 3 (Existence and stability of steady states under constant intervention: $\Upsilon = \pm\theta$). *Let E_1, E_2, E_3 , be the steady states as in Lemma (1). Consider intervention operations such that for $x > 0$, the authorities conduct sterilized sales of foreign bonds of size θ , so that $\Upsilon = -\theta$. Similarly, for $x < 0$, $\Upsilon = \theta$. Then, as the size of interventions increase, the non-fundamental steady states are drawn towards the fundamental equilibrium. I.e. As $|\theta| \rightarrow \infty$, $|x^*| \rightarrow 0$, and furthermore, for large enough θ , the non-fundamental steady states E_2, E_3 become stable.*

Lemma (3) shows how interventions operations reduce exchange rate fluctuations by firstly shifting the non-fundamental steady states closer toward the fundamental, and secondly, reducing the amplitude of fluctuations about the non-fundamental steady states.

To see the intuition behind this result, consider the implications of the exchange rate dynamics described by the model. Observed deviations in exchange rates from economic fundamentals are due to the evolutionary selection of trading strategies by market participants. In such a setting, intervention can only be effective in reducing persistent deviations from equilibrium if it can increase the proportion of traders employing strategies which forecast a move towards the fundamental equilibrium. Upon examination of the system (36) - (39), the role of intervention in the model becomes clear: *strong interventions, indicated by a large value of θ , reduce the realized profits from chartist strategies and result in a larger ratio of fundamentalists to chartists in the market.* Increasing θ represents the authorities selling overvalued currencies more vigorously - thereby inflicting losses on chartists who have been pushing the currency up. As the chartist rules become less profitable, the proportion of traders selecting those rules declines, and the exchange rate moves back toward equilibrium.

Intervention has an equivalent effect to a reduction in traders' sensitivity to differences in profits between strategies. So that under intervention, the mass of traders on the most profitable strategy at any point in time is reduced. Thus intervention reduces the amplitude of periodic fluctuations about unstable steady states, reducing short and medium run volatility. In addition, interventions draw the non-fundamental steady states closer to the fundamental. Both of these effects are illustrated in figure (4).

6.2 The Magnitude of Effective Interventions

If sterilized intervention operations by the monetary authorities represented the only source of change in the supplies of domestic and foreign outside assets, Rather, the authorities can exploit the processes governing the endogenous collapse of exchange rate bubbles to increase the efficiency in intervention operations. Indeed, if the goal of the authorities is to maintain the exchange rate within a band or target zone

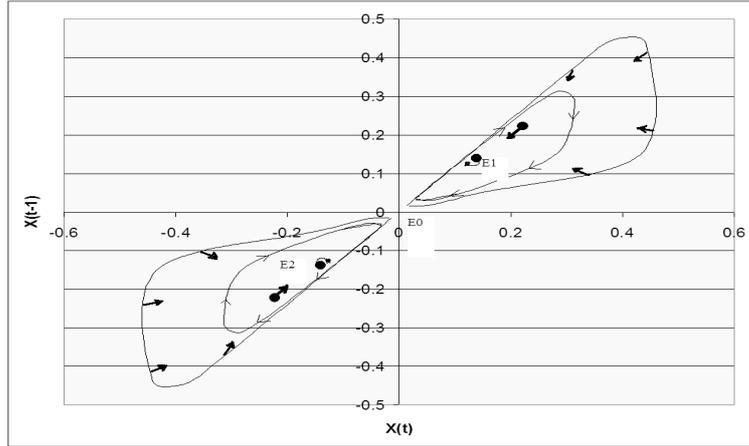


Figure 4: The effect of intervention on the non-fundamental steady states. Stronger interventions draw E_1 and E_2 closer to the fundamental, and reduce the amplitude of associated limit cycles.

about equilibrium, all that is needed is a series of one-off interventions of sufficient size when the exchange rate reaches the edge of targeted band.

To see why this is the case, recall the discussion in the previous section which highlighted a mechanism for the endogenous collapse of exchange rate bubbles. This is due to the proportion of traders employing chartist strategies decreasing as the deviation from the fundamental equilibrium increases, since the expected loss from employing a chartist strategy in the event of a collapse in the bubble rises with the deviation from equilibrium. The bubble bursts (endogenously) when the weight of traders on the chartist strategy is no longer sufficient to maintain its growth. Thus, in order to maintain the exchange rate within a desired band all that is required is for intervention to be large enough to effect a temporary shift towards fundamentals, after which the mechanism governing *endogenous* collapse of a bubble will take hold, returning the exchange rate to equilibrium.

A further implication of this is that, since the proportion of traders employing the chartist strategy is falling as the deviation from equilibrium increases, relatively smaller interventions are required for the weight of traders using the fundamentalist strategy to reach the 'critical mass' necessary for the bubble to collapse. Essentially, since when the deviation from equilibrium is large there are relatively few traders still employing the chartist strategy, the authorities need to turn fewer traders to the fundamentalist strategy to facilitate a return to equilibrium of the exchange rate. Alternatively, *interventions of a fixed size become more effective the greater the deviation from equilibrium*. This effect is illustrated in the model without noise in figure (5) below.

This feature of the model provides a rationale for the infrequent nature of interven-

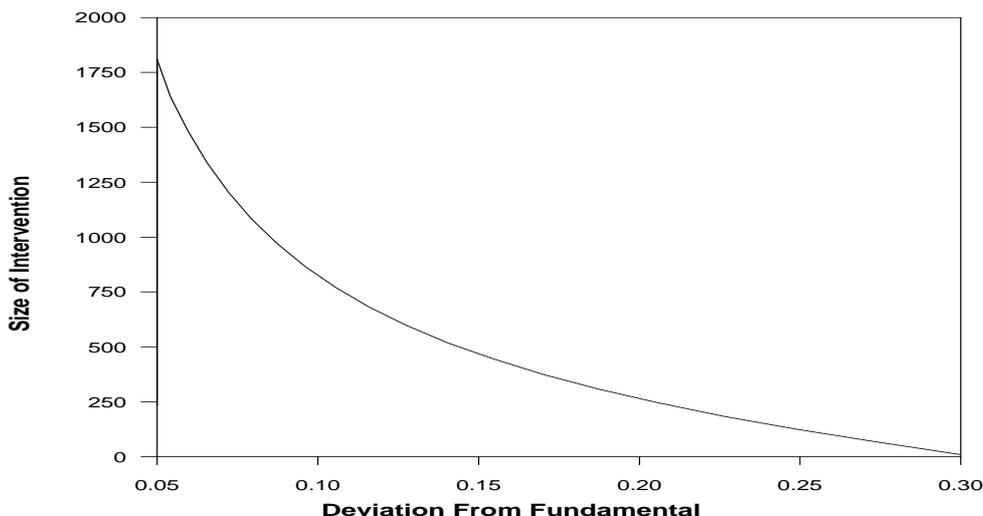


Figure 5: Absolute Value of Intervention Necessary to Create Turning Point

tion operations conducted by monetary authorities. The cost of effective interventions when the exchange rate is close to equilibrium is prohibitively high, and welfare losses associated with poor competitiveness are only likely to become a significant problem when the exchange rate persists at large deviations from fundamentals. Thus interventions are likely most cost effective when triggered by the exchange rate passing through a predetermined band.

In the next section, we turn to empirical testing and estimation of the model in the context of the Japanese experience with sterilized intervention in the 1990's. In addition, we shall also examine the influence of shocks to the exchange rate on the efficiency of intervention operations.

7 Empirical Evidence

This section presents empirical evidence in support of the model. Firstly, the literature on the effectiveness of sterilized intervention is briefly reviewed, as is the evidence regarding the profitability of technical analysis and sterilized intervention. The model is then tested using twelve years of Japanese data, and the implications for exchange rate dynamics and trader behavior discussed. Finally, a bootstrap procedure for the calculation of the probability of intervention being effective is presented, and the implications for the efficiency of intervention policy examined.

There is a large literature which empirically examines the effectiveness of sterilized intervention, growing in particular since the 1980's. Empirical work designed to test the channel through which intervention is effective has met with mixed success.

Regarding the portfolio balance channel, because of the small size of interventions relative to the outstanding stocks of bonds, most authors (e.g. Rogoff (1984)) have been skeptical that intervention could have a large impact. Indeed, many studies do not find evidence to support this channel, and those that do, such as Dominguez and Frankel (1993b), Evans and Lyons (2001) and Ghosh (1992), suggest that it is weak. The literature assessing the importance of the signalling channel has also, as yet, failed to reach a consensus regarding its importance. While Dominguez and Frankel (1993b) find a stronger effect of the signalling channel than that of the portfolio balance channel, in order for this channel to have an on going effect, the central bank must be seen to follow through with appropriate changes in monetary policy. However, Fatum and Hutchinson (1999) are unable to find an explicit link between intervention and future monetary policy, while Lewis (1995) and Kaminsky and Lewis (1996) suggest it occasionally operates in the wrong direction.

More recently, a number of studies have concluded that intervention can indeed be effective, but which leave the question of the channel of influence open. These papers typically employ an event study or case study approach. See, for example, Edison (1998), Fatum (2002), and Fatum and Hutchinson (2003). Of particular relevance to the study at hand is work by Ito (2002), which examines the Japanese experience with intervention in the 1990's. Ito finds that Japanese interventions were successful in producing intended effects on the yen during the second half of the 1990s.

Not only is there considerable evidence of the widespread use of technical trading strategies in the foreign exchange markets⁷, there is also a growing body of literature demonstrating its profitability⁸. In the model presented here, excess trading rule returns are explained by the presence of traders who employ the 'wrong' strategy. However, many authors have speculated that trading rule profitability may in practice be a direct result of intervention by the monetary authorities (e.g. Dooly and Schafer (1983), Corrado and Taylor (1986), and Sweeney (1986)). Furthermore, a number of recent studies have found that there is a high degree of correlation between interventions and trading rule returns (for example, LeBaron (1993) and Szakmary and Mathur (1997)). However, in contrast to previous studies, Neely (2002) has directly tested whether the timing of intervention and returns supports the hypothesis that intervention generates trading rule profits. He finds that in practice, abnormally high trading rule returns actually precede intervention operations, and rejects the hypothesis that they are generated by intervention. This lends support to a central result of the model presented in section (5): that intervention by the authorities can reduce misalignments by reducing the profitability of destabilizing trading rules.

Indeed, for intervention to be effective, it must be the case that the monetary au-

⁷The literature of the use of trading strategies in the foreign exchange markets is surveyed in section (2)

⁸See, inter alia, Levich and Thomas (1993), Neely, Weller, and Dittmar (1997), Osler and Chang (1999), and Sweeney (1986).

thorities buy when the currency is low and sell when it is high, in which case they themselves should make profits from intervention operations. Ito (2002) finds that the Japanese interventions earned close to 9 trillion yen during the 1990s from realized capital gains, unrealized capital gains, and interest rate differentials. He thus concludes that the Japanese authorities were a successful, and thus stabilizing, speculator.

7.1 Data and Model Estimation

In this section the model is estimated using monthly Japanese data from January 1991 to April 2003. In July 2001 the Japanese Ministry of Finance (MOF) became the first government agency to release all data on sterilized intervention operations to the public⁹, which provides a useful and timely test-bed for the model. As this data shows, the Japanese monetary authorities have intervened frequently, especially in the dollar/yen market, and also occasionally in the yen/euro (yen/mark) market.¹⁰ The Japanese experience therefore provides a wealth of data, making it ideally suited to empirical analysis.

In any study of exchange rate 'misalignments', the model employed to calculate the equilibrium exchange rate is of central importance. In line with the model presented in section 2, the estimated deviations from rational expectations equilibrium, \hat{x}_t , are calculated as the residuals from an OLS regression of the spot yen/dollar rate on a constant and monetary fundamentals:

$$s_t = \mu + \alpha((m_t - y_t) - (m_t^* - y_t^*)) + x_t \quad (40)$$

where s_t is the log of the spot yen/dollar exchange rate, m_t domestic (Japanese) M1, and y_t the log of the domestic industrial production index. Starred variables indicate similar quantities for the US. Monthly data for the spot Yen/Dollar exchange rate are taken from the IMF's International Financial statistics database, as are the data for US M1. Industrial production indices are used as a proxy for aggregate demand in the Japan and the US, and these series are also taken from the IFS database. Japanese M1 is taken as a measure of Japanese money supply, with the data obtained from the Bank of Japan's (BOJ) website, whilst data on the Japanese sterilized intervention operations are taken from the MOF website. Figure (6) below illustrates the estimated equilibrium exchange rate together with the actual spot rate over the period under study.

⁹An analysis of the aims and efficiency of these intervention operations is conducted in a linear framework by Ito (2002).

¹⁰This is in stark contrast to the US authorities, which since 1995 have intervened only once in the dollar/euro market (in 2001), and have not not intervened in the yen/dollar market at all.

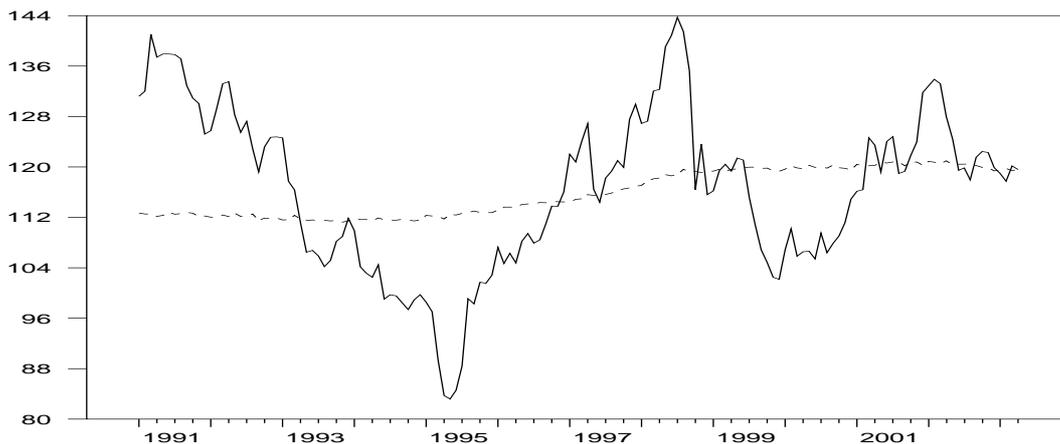


Figure 6: Spot yen/dollar exchange rate and estimated equilibrium rate

The estimated value of the coefficients in the above model, were $\hat{\mu} = 2.6837$, and $\hat{\alpha} = 0.1562$, with t-statistics of (4.5560) and (3.5115) respectively¹¹, providing strong evidence in favor of the model at the 95 percent significance level.

Clearly, more sophisticated approaches to the estimation of the equilibrium exchange rate could be employed to allow for the influence of factors such as productivity differentials (along the lines of the Balassa-Samuelson effect (see, for example, Froot and Rogoff (1995)), or the size of current account deficits (as, for example in the FEER suggested by Williamson (1984)). However, the simple monetary model-based specification used here does seem to produce plausible estimates for the equilibrium rate. PPP based estimates for the yen/dollar rate typically produce relatively high values for the dollar, or the order of 150-170 yen, while FEER estimates are often much lower (90-100 yen) (e.g. Wren-Lewis (2000)). The estimates used in this paper all range between 111 and 120 yen to the dollar. For example, the estimated equilibrium exchange rate reaches a minimum of 111.2 yen to the dollar in November 1993, and returned to a level of 119.4 yen to the dollar in December 2002.

Graphs of the deviation of the exchange rate from equilibrium implied by monetary fundamentals, \hat{x}_t , together with sterilized monthly sales of Yen (in billions of Yen) by the Japanese authorities are given in figure (7).

A general specification for testing the model is

$$x_t = \phi_0 Int_t + \sum_{j=1}^p \phi_j x_{t-j} - \sum_{j=1}^p \phi_j x_{t-j} \Phi(\cdot) + \epsilon_t \quad (41)$$

¹¹Due to the non-stationarity of the fundamental variables, t-statistics were calculated with a bootstrap procedure based on 10000 simulations of the estimated model.

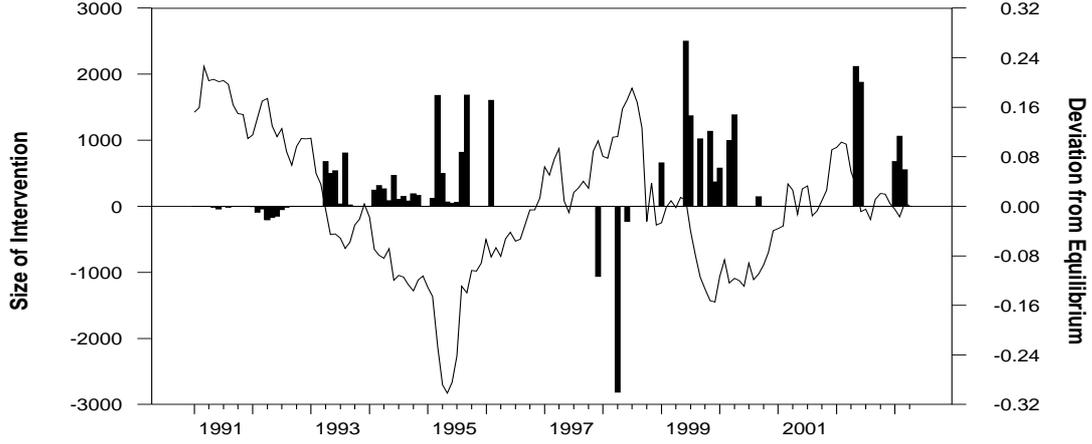


Figure 7: Interventions and Deviation from Equilibrium

where $\epsilon_t \sim iid(0, \sigma^2)$. The transition function, $\Phi(\cdot)$, maybe written in reduced form as

$$\Phi(\cdot) = [1 + \exp(-\gamma(-\beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_4 x_{t-3} - \beta_5 Int_t)x_{t-3} + c)]^{-1} \quad (42)$$

It is worth pointing out that the model provides a market based rationale for the application of Smooth Transition Regression Models (STR) suggested by Granger and Teräsvirta (1993), which have recently become popular in modelling nonlinearities in exchange rates. The function $\Phi(\cdot)$ maybe recognized as an (albeit complicated) form of logistic transition function used in LSTAR models.

Following Granger and Teräsvirta (1993), an LM test for linearity with optimal power against the type of nonlinearity implied by the model employs a three step procedure based on a first-order Taylor approximation of the transition function around about $\gamma = 0$.

1. Estimate the linear model

$$x_t = \phi_0 Int_t + \sum_{j=1}^p \phi_j x_{t-j} + u_t \quad (43)$$

and save the residuals \hat{u}_t . Define $SSR_0 = \sum \hat{u}_t^2$.

2. Use these residuals to run the regression

$$\hat{u}_t = \psi_0 Int_t + \sum_{j=1}^p \psi_{1j} x_{t-j} + \sum_{j=1}^p x_{t-j} (\psi_{2j} x_{t-1} + \psi_{3j} x_{t-2} + \varphi_j Int_t) x_{t-3} + v_t \quad (44)$$

and save the residuals \hat{v}_t . Define $SSR_0 = \sum \hat{v}_t^2$.

3. Compute the test statistic $LM = T(SSR_0 - SSR_1)/SSR_0$, where T is the number of observations. Under the null hypothesis, LM is distributed $\chi^2(pk)$. In small samples, the equivalent F statistic is $F = [(SSR_0 - SSR_1)/pk]/[SSR_0/(T - (2pk + 1))]$.

Proceeding in this manner yields an F -statistic of 32.59, so that the null hypothesis of the true model being linear is easily rejected at the 95% significance level. The test thus provides strong support for the type of non-linearity implied by the theoretical model.

A problem frequently encountered in the literature assessing the impact of intervention is that of simultaneity bias. Many studies that do not take account of simultaneity bias find that the estimated coefficients on asset supplies are statistically significant, but enter with the wrong sign. The problem arises because exchange rates and asset supplies may be simultaneously determined. For example, a domestic currency appreciation may result in a change in the relative supplies of domestic and foreign assets through current account imbalances, or through endogenous market intervention. In order to allow for possible simultaneity bias between the exchange rate and intervention operations the model was estimated using nonlinear instrumental variables¹².

Strictly speaking, the theory outlined in section (4.3) requires that only the stock of intervention can influence the risk premium. However, the flow of intervention may also influence the risk premium if the rate of adjustment of asset stocks held is proportional to the difference between desired asset stocks (a function of the risk premium), and the actual holdings of asset stocks (Dominguez and Frankel (1993b)). For this reason, the model was estimated using both total cumulated intervention and intervention flows.

The final parameter estimates, together with bootstrapped confidence intervals¹³, are given below.

¹²The instruments used were $x_{t-1} \dots x_{t-6}$ and $Int_{t-1}x_{t-3} \dots Int_{t-6}x_{t-9}$.

¹³Since the path of the exchange rate is explosive when a large enough proportion of traders employ the chartist strategy, standard t -statistics are invalid. Reported bootstrapped confidence intervals are based on 1000 replications of the model.

Parameter	Estimate	95% Confidence Interval	
ϕ_0	2.62×10^{-6}	-8.23×10^{-07}	$\pm 1.1576 \times 10^{-05}$
ϕ_1	1.1968	1.1927	± 0.0686
ϕ_2	0.4650	0.4699	± 0.0878
β_1	-8474.2600	-15012.6000	± 7712.7880
β_2	-10767.3000	-21038.7000	± 10994.7400
β_3	1712.4700	1093.6470	± 2367.7020
β_4	-3.5227	-4.8025	± 2.9740
c	-97.5961	-163.6640	± 92.8260

$$x_t = 2.62 \times 10^{-6} Int_t + 1.1968x_{t-1} - 0.4650x_{t-2} - [1 + \exp(-(-8474.26x_{t-1} + 10767.30x_{t-2} + 1712.47x_{t-3} - 3.52Int_t)x_{t-3} + 92.83)]^{-1} \quad (45)$$

The estimation results provide strong evidence of nonlinear behavior consistent with the theoretical model. The parameter estimates $\hat{\phi}_1$ and $\hat{\phi}_2$ indicate that the endogenous fluctuations in the exchange rate move away from the fundamental equilibrium at a rate of 18.04% per month when all traders employ the chartist strategy, and towards fundamentals at a rate of 20.82% when all traders follow the fundamentalist strategy. Furthermore, large values of the estimates of β_1 , β_2 , β_3 , and c indicate that switches between the extreme cases where where all traders employ either one of the available strategies occur rapidly, so that the dynamics of the exchange rate can be classified as belonging to either an explosive regime or a strongly mean reverting regime, depending on the strategy employed. The regime classification suggested by the model is illustrated in figure (8).

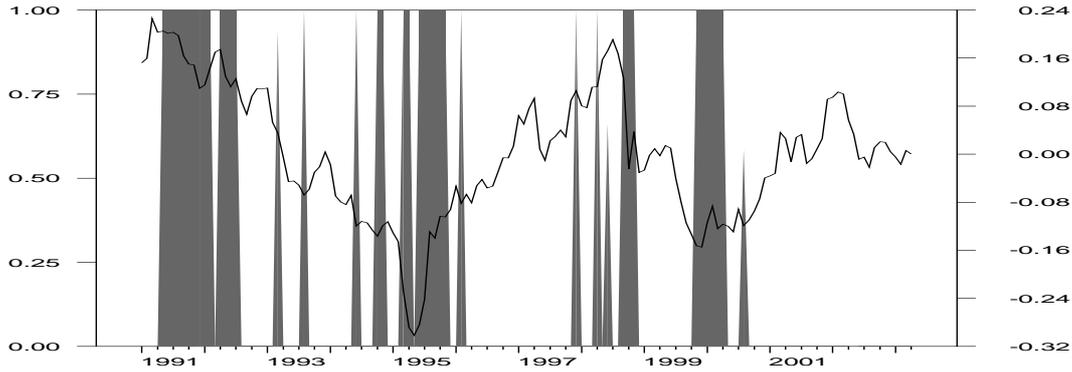


Figure 8: Regime Identification. The estimated proportion of traders employing the fundamentalist strategy is indicated by grey shading (left scale).

In the absence of noise the estimation results imply that the true value of the intensity of choice is large enough that, consistent with Lemma 1, the steady state

corresponding to the fundamental equilibrium is unstable, and that the exchange rate oscillates about two (unstable) non-fundamental steady states. The results suggest the exchange rate follows a pair of limit cycles about the non-fundamental steady states, thus giving further empirical support to the analysis presented in section (5). Were there to be no shocks to the exchange rate, the parameter estimates indicate the exchange rate would evolve according to a pair of limit cycles as illustrated in figure (9).

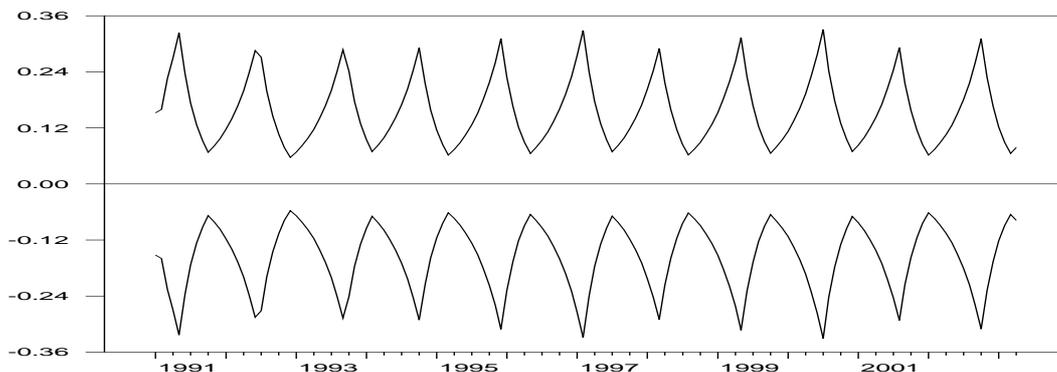


Figure 9: Estimated Limit Cycles

The parameter d represents the lag with which intervention influences the exchange rate dynamics, and a value of $d = 0$ was chosen on the basis of likelihood ratio tests. Consistent with the consensus from previous studies¹⁴, the linear effect of interventions on the equilibrium exchange rate is small and insignificantly different from zero. However, the data strongly support the models' prediction that interventions may have a strong non-linear effect on deviations of the exchange rate from fundamentals¹⁵. Recall that the mechanism by which the nonlinear effect of intervention can significantly influence the exchange rate dynamics, while the linear effect is negligible, is due to a high degree of sensitivity amongst traders of differences in profitability between the available strategies. I.e. the strong nonlinear and weak linear effects of intervention are primarily due to the large estimate of the intensity of choice implied by the parameter estimates.

The finding that only intervention flows have a significant effect on the exchange rate are consistent with the common reports in the literature that the effects of sterilized intervention are typically short-lived. However, while the initial impact of intervention is limited to the short run the highly nonlinear dynamics of the exchange rate, and resultant path-dependance, result in the actual effects of intervention on the exchange rate being highly persistent. This effect is discussed in more detail in

¹⁴See, inter alia, Dooley and Isard (1982), Dooley and Isard (1983), Frankel (1982), and Rogoff (1984).

¹⁵While the results from the model using intervention flows suggested strong effects on the exchange rate, the coefficient on cumulated intervention was not significant, and for this reason are not reported here.

section (8).

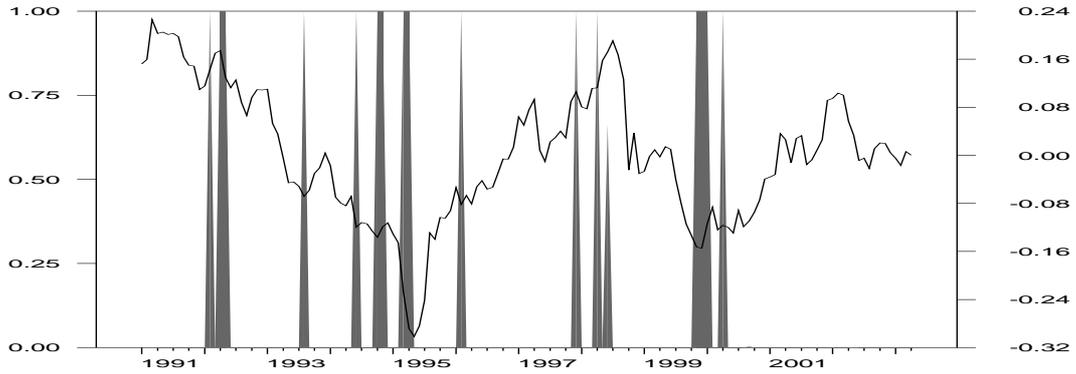


Figure 10: Switches to fundamentalist strategy caused by intervention operations.

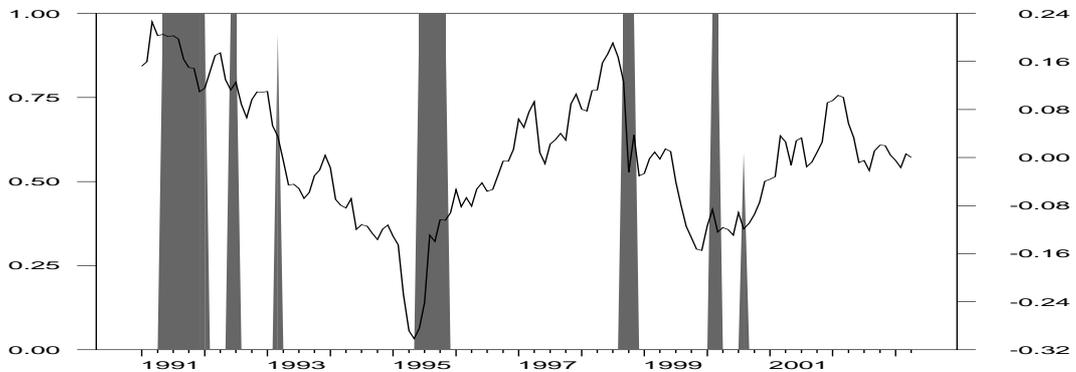


Figure 11: Endogenous switches to the fundamentalist strategy.

Figure (10) shows the instances in which intervention operations resulted in switches from the chartist to the fundamentalist strategy by traders, while figure (11), illustrates cases where traders switch from the chartist to the fundamentalist strategy occurred independently of such interventions. As is clear from the figures, a large proportion (estimated to be 32%) of periods when traders employed fundamentalist strategies can be attributed to the use of intervention operations by the authorities. However, there are also a large number of intervention operations which apparently failed to result in a shift from chartist to fundamentalist strategies by market participants. Clearly, an important question for the monetary authorities is how large should interventions be in order to instigate a movement toward equilibrium in the exchange rate? In the next section, this question is addressed directly, along with the implications of the results for the conduct of exchange rate policy.

8 The Efficiency of Intervention Policy

In this section it is shown how the model has strong implications for exchange rate policy designed to limit market induced misalignments. Firstly we present a simulation procedure for computing confidence intervals for intervention necessary to reverse misalignments. As will be seen, the results provide a new source of ammunition to the case for exchange rate management based on 'target zones'. The discussion then moves to an assessment of the efficiency of intervention policy in Japan during the 1990's. It is shown how the estimation results imply that, consistent with the findings of ?, interventions under Eisuke Sakakibara were particularly effective. Finally, efficacy of intervention implied by the model is further emphasised with a discussion of the counter-factual exchange rate that would have prevailed had the authorities failed to intervene.

The discussion in section (6.2) identified how, in the deterministic model, it is possible for the monetary authorities to prick bubbles in the exchange rate with a one-off intervention of sufficient size. Furthermore, evidence from simulations suggested that the necessary size of interventions decreases as the size of the misalignment grows.

When moving to a stochastic environment however, a number of complications arise. In particular, the size and sign of exogenous shocks to the exchange rate will have important consequences for the efficiency of interventions. For example, a shock which reinforces the trend in the exchange rate away from fundamentals, and which occurs in the same period as the intervention could easily nullify its effects if it is of sufficient size. Conversely, a shock which moves the exchange rate toward equilibrium would result in a smaller than usual intervention being necessary to achieve the desired objective. Under the assumption that the authorities cannot observe the size and sign of shocks before intervening, in this section a bootstrap procedure is presented for the calculation of confidence intervals for the size of effective interventions (a detailed description of the procedure is given in the appendix).

The model is assumed known, so that sample variability is not taken into account. Shocks for periods 0 through s are drawn with replacement from the residuals of the estimated model and, for a given initial history of the deviation x , fed through the estimated model to produce a simulated data series. The data series thus constructed is a forecast of the exchange rate conditional on the set of initial values and a given sequence of shocks. Next, the same procedure is followed except that a single intervention operation of size θ is included once the absolute value of x has reached a given level δ . The point of intervention is fixed such that intervention occurs only when the exchange rate is following a trend leading away from equilibrium, and when the trend would be sustained in the absence of intervention (i.e. the baseline forecast is moving away from equilibrium). The intervention is deemed to be effective if it results in a movement in the exchange rate toward equilibrium,

i.e. $|x_\tau| < |x_{\tau-1}|$, where the intervention occurs at time $t = \tau$. Conditional on the initial history, sequence of shocks, and threshold at which intervention occurs, the smallest value of intervention which is effective in moving the exchange rate towards equilibrium is then determined by a grid search. Levels of effective intervention are computed in this way for 1000 draws from the residuals to give a distribution conditional on the initial history and intervention threshold. The process is repeated for 30 different initial histories of x , and the results averaged to produce a distribution of effective intervention conditional only on the intervention threshold. Figure (12) shows how the 25, 60 and 70 percentiles of the distributions thus calculated vary as the intervention threshold changes. They give an approximate method of determining the probability that total monthly intervention operations of a given size, carried out at a given deviation of the exchange rate \bar{x} , will be effective.

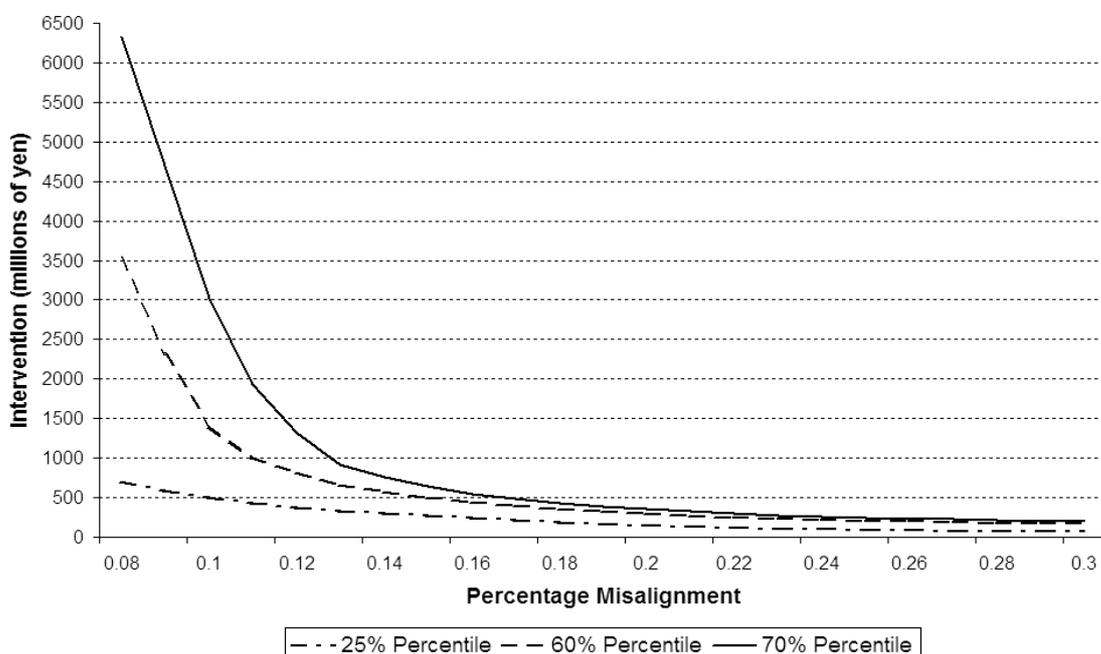


Figure 12: Probability of Interventions Effectively Inducing a Turning Point

For example, in order to have a 70 percent chance of moving the exchange rate toward fundamentals when the deviation from equilibrium is 13 percent, interventions of approximately 1 trillion yen are required. To achieve the same probability of mean reversion at a deviation from equilibrium of 9 percent requires interventions of around 5 trillion yen per month. However, within the sample there were only 13 months in which total interventions exceeded 1 trillion yen, and the largest total intervention in a single month was of the order of 2.8 trillion yen.

Qualitatively, these findings suggest that sterilized intervention can be used with greater efficiency when the degree of misalignment is large. Furthermore, the quantitative results indicate that the efficiency of intervention decreases exponentially as

the degree of misalignment falls. For example, the empirical findings above show that to achieve a reasonable probability of mean reversion in the exchange rate when closer to equilibrium than say 12 percent, the size of intervention needs to be so large that it is unfeasible. These predictions have strong implications for the exchange rate policy regime (or lack of) adopted by the G-3. The next section examines this issue in more detail.

8.1 Implications for the choice of exchange rate regime

The growing evidence that sterilized intervention can indeed be effective, together with the finding that intervention is much more successful when misalignments are large, has strong implications for exchange rate management regimes based on sterilized intervention in a world with free movement of capital.

Firstly, the finding that sterilized intervention can have strong effects on misalignments, but negligible effects on the equilibrium rate is in-keeping with the received wisdom that a country cannot maintain an exchange rate policy wholly independent of other policy choices. Instead, the role of intervention is likely limited to maintaining exchange rates at levels broadly in keeping with underlying economic conditions. Secondly, the large gains in efficiency of intervention when misalignment are large provides further ammunition to the case for intermediate exchange rate regimes based on exchange rate bands. Clearly, the quantity of intervention required to keep the exchange rate within a band falls rapidly as the width of the band increases. More importantly perhaps, the size of intervention necessary to significantly influence the exchange rate when it is close to equilibrium is so large as to make such a policy unworkable in practice.

A key feature of much of the discussion surrounding the likely success of a target zone is the focus on the credibility of the authorities' commitment. For example, Krugman and Miller (1993) argue that traders' speculation shifts from being destabilizing to stabilizing as long as they are assured that the target zone will be maintained. Their argument focuses on the role of stop-loss traders, who cover their exposure to large losses by selling their assets when prices fall below a certain level. If a target zone commands sufficient credibility to assure such traders that their stop-loss orders will not be triggered, it removes the fuel for destabilizing speculation. A similar case is put forward by Jeanne and Rose (2002), who suggest that a sufficiently credible commitment to keep the volatility of the exchange rate within a preannounced range can prevent destabilizing speculators entering the market. This effect is achieved by lowering the risk premium on foreign bonds which removes the high excess returns on foreign assets which lure destabilizing speculators in the first place. Note however, that the results discussed here suggest that an exchange rate band could still prove effective even when lacking full credibility. While an even partially credible announcement to intervene does improve the effectiveness of inter-

vention in the model presented here¹⁶ it is not necessary to reduce the attractiveness of chartist strategies. This is because, by its very nature, chartist behavior is backward looking. Intervention which reduces the profitability of technical rules in one period therefore affects the strategy choice the next.

What kind of exchange rate band would be most appropriate given these results? A key implication of the use of *sterilized* intervention is that even massive interventions may not be immediately successful in reversing destabilizing trends. Given this observation, and the well documented susceptibility to speculative attack of target zones where the authorities are committed to defending the edges of the band, the most appropriate arrangement would seem to be the 'Monitoring Exchange Rate Band' proposed by the Tarapore committee and discussed by Williamson (1998). In a monitoring band, the authorities announce their intention to maintain the exchange rate within a band, but are not committed to intervene when the edge of the band is reached. Thus there is a public threat that intervention may be triggered once the exchange rate reaches a preannounced deviation from fundamentals, whereafter interventions continue until the exchange rate is brought back inside the band. This allows for flexibility in the face of overwhelming market pressure, and removes the threat of speculative attack.

This kind of regime would take full advantage of the increased efficiency of intervention at large misalignments. The results presented here demonstrate that, since interventions of a practical size only have a reasonable change of success outside a region of around 12 percent around fundamentals, a monitoring band policy is exactly what is called for in order to place realistic limits on the size of persistent exchange rate fluctuations. There is also no requirement for the announced band to be fully credible. Prolonged misalignments are generated when traders find forecasts based on past movements in the exchange rate more profitable than forecasts based on underlying economic conditions. Intervention can be successful because it reverses this situation. Indeed, this is exemplified by the Japanese experience: since intervention proved quite successful despite the absence of a announced band for the exchange rate.

8.2 The efficiency of Japanese intervention operations in the 1990's

The distributions illustrated in figure (12) may be used to calculate the ex-ante probability that actual intervention operations by the Japanese authorities would result

¹⁶An announcement by the authorities to intervene improves interventions effectiveness when market participants condition their choice of forecasting strategy on the likelihood of intervention. They do so because they are aware that intervention will reduce the profitability of chartist strategies.

in a reversal of trend in the exchange rate¹⁷ These probabilities are illustrated in figure (13), together with actual intervention operations and the estimated deviation from fundamentals when the intervention took place.

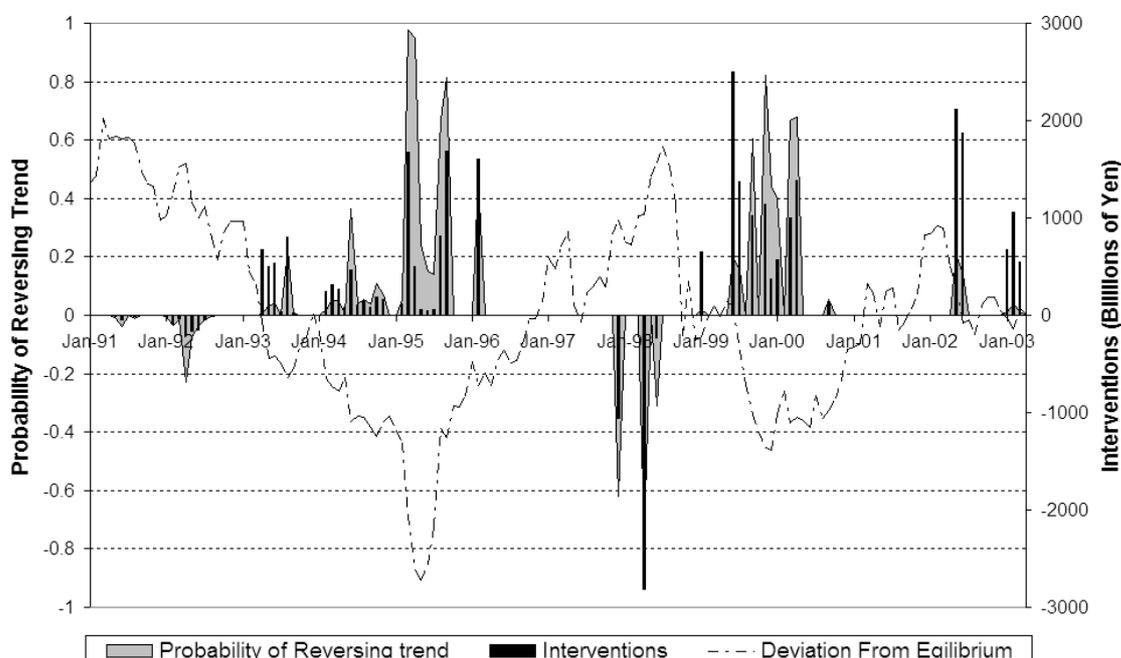


Figure 13: Probability of Actual Interventions Inducing Turning Points

The figure highlights how the efficiency of interventions by the Japanese monetary authorities has varied with the period in which they occurred. In particular, it lends further support to the results of Ito (2002), who finds that interventions conducted when Eisuke Sakakibara was Director General of the International Finance Bureau at the MOF. Referring to his book (Sakakibara (2000)), Ito details how intervention policy under Sakakibara was distinctively different from that of his predecessor. Specifically, interventions became much less frequent, and were considerably larger than before. Ito's results indicate that, in contrast to his predecessor, intervention under Sakakibara was successful in producing the authorities desired effects on the yen. Figure (13) illustrates a sharp contrast in the probability of intervention being effective between the periods when Sakakibara was in office at the MOF from 1995 to 1999, and the periods prior to him taking up his position and after he had left. For example, the interventions carried out during September 1995, when both the estimated deviation from fundamentals and the size of intervention were large, are estimated to have a relatively large probability of success (approximately 80%). Prior to 1995, the probability of intervention being effective never exceeded 38%,

¹⁷While the analysis presented here focuses on the probability that intervention will *reverse* the trend in the exchange rate, there is also evidence (e.g. Ito (2002)) that intervention is used to reinforce stabilizing trends in the exchange rate. Clearly this type of intervention could be considerably smaller and yet remain effective.

and was often considerably smaller. Similarly, the results indicate that interventions after April 2000 have had less than 20% chance of changing the trend in the exchange rate.

It is interesting to note that the model identifies many interventions as having a significant effect on the path of the exchange rate, even in case where they do not seem to effect the trend in the short run. These can be interpreted as instances when, although interventions apparently failed to 'burst bubbles', the movement of the exchange rate away from fundamentals would have been more pronounced in their absence. This is because even if the intervention fails to prompt enough traders to switch to the fundamentalist strategy to burst the bubble, it will nevertheless result in a reduction of the mass of traders on the chartist strategy, thereby reducing the force of the trend away from equilibrium. Furthermore, even if the intervention fails to result in the exchange rate bubble bursting immediately, it may still prove effective in prompting a medium-run collapse of the bubble at significantly lower levels of the exchange rate than would have been the case had the authorities not intervened. To see why this is the case, recall that in the medium run the proportion of traders employing the chartist strategy inevitably falls as the exchange rate moves further from equilibrium. An intervention operation by the authorities has the effect of reducing still further the proportion of traders using the chartist rule. Although the intervention will only be successful in bursting the bubble if it is large enough to prompt enough traders to switch to the fundamentalist strategy to result in a move toward equilibrium in the exchange rate. However, even if the size of the intervention is insufficient to result in the bubble bursting right away, the weight of traders using the chartist strategy will still be somewhat reduced, which will have two key consequences. Firstly, the rate at which the exchange rate is moving away from equilibrium may be substantially reduced. Secondly, if the reduction in the mass of traders on the chartist strategy persists (i.e. traders who switched to the fundamentalist strategy following the intervention do not switch back to the chartist strategy one the authorities stop intervening), then, as the bubble grows further, a relatively smaller proportion of traders is required to endogenously switch to the fundamentalist strategy in order for the bubble to collapse. This implies that even if the intervention is unsuccessful in the short run, it may well result in the collapse of the bubble in the medium run at substantially smaller deviations from fundamentals than would have been the case had the authorities failed to intervene.

This delayed effect of intervention may be clarified, and a fuller picture of the actual effect of intervention operations gained, by simulating the path of the exchange rate subject to the realized sequence of shocks in the absence of intervention by the authorities. The simulated path of the exchange rate in the absence of intervention is compared with the actual path in figure (14).

Figure (14) clearly illustrates how intervention by the Japanese authorities has been successful in reducing medium run volatility in the exchange rate. While many

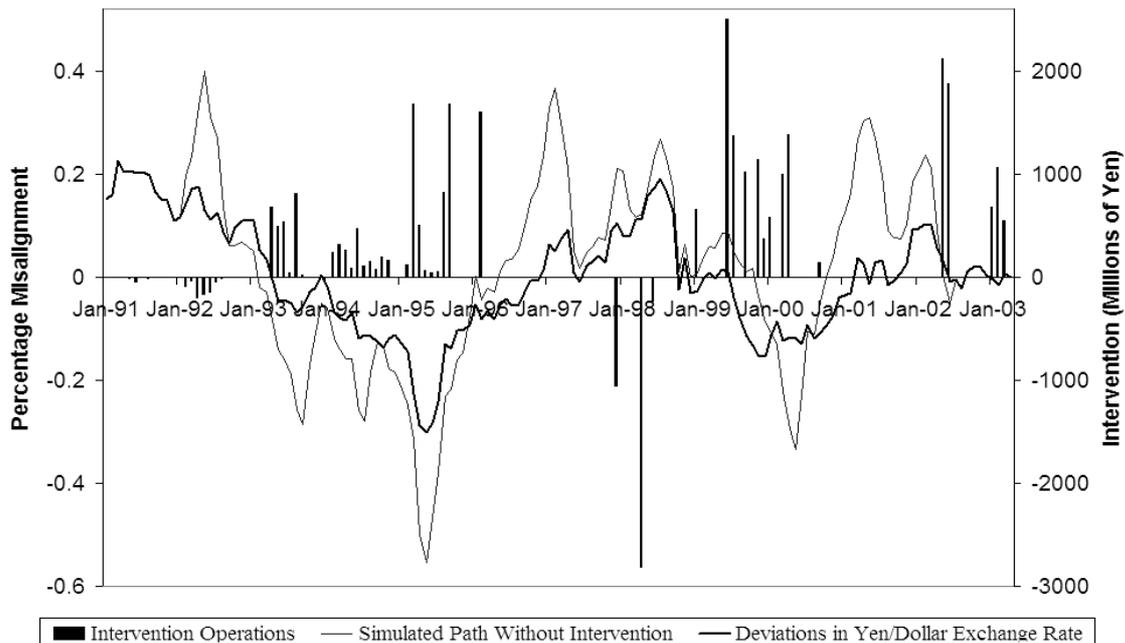


Figure 14: Simulated Path of Exchange Rate in Absence of Intervention

interventions apparently failed to *reverse* the trend of the exchange rate away from equilibrium, the figure illustrates how they were effective in *reducing the strength* of the trend. Intervention by the authorities thus seems to have been highly successful in limiting erratic and persistent movements in the exchange rate created by herding behaviour amongst market participants.

9 Conclusion

Recent work on the presence of noise traders and chartists in the foreign exchange market suggests that the large and persistent swings evident in G3 exchange rates have little relation to movements in economic fundamentals. In order to limit the negative effects of these misalignments on competitiveness, inflation, and financial stability, foreign exchange intervention by, for example, the Japanese monetary authorities, is often used to reduce the severity of such swings. The central hypothesis of this paper is that such interventions can be successful by reducing the profitability of destabilizing trading strategies, thereby inducing market participants to base their forecasts on economic fundamentals. It has been shown how even when interventions are sporadic in nature they can significantly alter the path of the exchange rate in the medium term, effectively bursting bubbles in the exchange rate.

Drawing on twelve years of data on Japanese interventions, there is significant em-

pirical support for the model. Even when intervention by the Japanese authorities has apparently failed to burst bubbles in the exchange rate in the short run, simulations of the model suggest that interventions have often led to the collapse of bubbles somewhat earlier than would have been the case in the absence of intervention. Furthermore, we have provided estimates of the ex ante probability that interventions of a given size will be effective in bursting bubbles. For these results to provide an operational guide to the authorities on the size actual interventions clearly requires modeling at daily or higher frequencies. However, they do provide an indication of the likely efficiency of interventions in the medium term.

The emphasis in this paper has been on limiting exchange rate *misalignments* brought about by the use of backward looking trading strategies. The exchange rate fluctuations studied are generated and corrected in an endogenous manner by traders' selection of forecasting rule. This provides an explanation for the market's own tendency to correct extreme misalignments. One implication of this framework is that it provides an explanation for the rather mixed evidence on the efficacy of intervention in the literature. It was found that traders' high sensitivity to differences in the profitability between competing strategies allows intervention to be effective in reducing misalignments, even when it is of insufficient size to generate portfolio balance effects capable of influencing the equilibrium exchange rate. It is possible that the failure of many studies to differentiate between these effects has resulted in indeterminate results.

From the perspective of the monetary authorities, the mechanisms underlying the type of misalignment studied here can be harnessed to increase the efficiency of intervention operations. At larger misalignments a greater proportion of traders base their forecasts on fundamentals. As a result, a smaller quantity of intervention is required to tip the balance and achieve a 'critical mass' of traders who forecast a shift in the exchange rate to levels in line with economic fundamentals. In this way, intervention is seen to become more efficient as the size of misalignment increases. On the other hand, this implies that the efficiency of sterilized intervention is greatly reduced close to equilibrium. Based on these results, one can conclude that the optimal policy regime for the use of sterilized intervention would resemble a target zone.

While this paper has focused on the role of heterogenous expectations in explaining exchange rate fluctuations and the role of sterilized intervention, an alternative explanation could draw on the role of asymmetric information between market participants¹⁸. Further work could assess the role of intervention in such a framework. This would also enable an assessment of the role of aural intervention by the authorities in controlling exchange rate fluctuations.

¹⁸See, for example Abreu and Brunnermeier (2003)

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Appendices

A Proofs of Lemmas

Proof of Lemma 1. From the market equilibrium equation (36), from the market equilibrium equation (36), that the only possible steady states occur when either $x^* = 0$, or

$$\frac{(1 + \alpha)}{\alpha} = g - \omega^* q \quad (46)$$

$$w^* = \frac{g}{q} - \frac{1 + \alpha}{q\alpha} \quad (47)$$

Now, we require that $0 \leq w^* \leq 1$, so that non-fundamental steady states exist only if $\omega^* < 1$, so that

$$\frac{(1 + \alpha)}{\alpha} > v, \quad (48)$$

which always holds. Also, if $\omega^* > 0$, then $\frac{g}{q} - \frac{1 + \alpha}{q\alpha} > 0$. I.e.

$$g > \frac{(1 + \alpha)}{\alpha} \quad (49)$$

When $\Upsilon = 0$, the steady state deviation from equilibrium, x^* , is given by the solutions (if they exist) to

$$\omega^* = \frac{1}{1 + \exp\left[\frac{\beta q}{a\sigma^2}(x^*)^2\left(\delta(g + v) - \frac{\delta(2\alpha + 1) + 1}{\alpha}\right) + \beta C\right]}, \quad (50)$$

in which case

$$\frac{\beta q}{a\sigma^2}(x^*)^2\left(\delta(g + v) - \frac{\delta(2\alpha + 1) + 1}{\alpha}\right) + \beta C = \ln\left[\frac{1 - \omega^*}{\omega^*}\right], \quad (51)$$

and

$$x^* = \pm \sqrt{\frac{\alpha a \sigma^2}{\beta q} \left(\frac{1}{1 + \delta(2\alpha + 1) - \alpha \delta(g + v)} \right) \left(\beta C - \ln\left[\frac{1 - \omega^*}{\omega^*}\right] \right)}. \quad (52)$$

Now, these values for the non fundamental steady states, x^* , are real if either

1. $(g + v) < 2 + \left(\frac{1 + \delta}{\alpha\delta}\right)$ and $\exp(\beta C) > \frac{1 - \omega^*}{\omega^*}$
- or
2. $(g + v) > 2 + \left(\frac{1 + \delta}{\alpha\delta}\right)$ and $\exp(\beta C) < \frac{1 - \omega^*}{\omega^*}$

Thus non-fundamental steady states exist if β is small and $g + v > 2 + \left(\frac{1 + \delta}{\alpha\delta}\right)$, or if β is large and $(g + v) < 2 + \left(\frac{1 + \delta}{\alpha\delta}\right)$.

Stability of the fundamental steady state is evaluated by considering the characteristic equation

$$\lambda^2 \left(\frac{\alpha}{(1 + \alpha)}g - \frac{\alpha}{(1 + \alpha)}q((1 + \exp[\beta c]^{-1}) - \lambda) \right) = 0 \quad (53)$$

with eigenvalues

$$\lambda_1 = 0, \quad \lambda_2 = 0, \quad \lambda_3 = \frac{\alpha}{(1 + \alpha)}g - \frac{\alpha}{(1 + \alpha)}q((1 + \exp[\beta c]^{-1}) \quad (54)$$

Note firstly that since $q = g - v$, $q < g$ and also $(1 + \exp[\beta c]^{-1}) \leq 1$, so that λ_3 must be real and positive.

Now, the fundamental steady state loses stability when the eigenvalue λ_3 crosses the unit circle.

$\lambda_3 > 1$ iff

$$\frac{\alpha}{(1 + \alpha)}g - \frac{\alpha}{(1 + \alpha)}q((1 + \exp[\beta c]^{-1}) > 1, \quad (55)$$

or

$$v + g \exp[\beta c] > \frac{(1 + \alpha)}{\alpha} + \frac{(1 + \alpha)}{\alpha} \exp[\beta c]. \quad (56)$$

Since $v < \frac{(1 + \alpha)}{\alpha}$ and $g > \frac{(1 + \alpha)}{\alpha}$, while for small β the LHS of (56) is less than the RHS, as β increases there must be some value whereafter (56) is satisfied. In general then, for $g > \frac{(1 + \alpha)}{\alpha}$, as β increases there exists some β^* for which $\lambda_3 = 1$, where a pitch-fork bifurcation leads to the creation of the two non-fundamental steady states. \square

Proof of Lemma 2. The characteristic equation for the non-fundamental steady states is

$$g(\lambda) = \lambda^3 - \left(1 + \frac{\alpha(1-\delta)}{(1+\alpha)}\zeta\right)\lambda^2 + (1+\delta)\zeta\lambda - \left(\frac{2\alpha\delta(g+v-1) - (1+\delta)}{(1+\alpha)}\right)\zeta \quad (57)$$

where

$$\zeta = \frac{\beta q^2}{a\sigma^2}(x^*)^2\omega^*(1-\omega^*). \quad (58)$$

At the pitchfork bifurcation value $\beta = \beta^*$, $x^* = 0$, so that $\zeta = 0$. For β slightly larger than β^* , ζ is slightly larger than 0 and (57) has three real eigenvalues inside the unit circle. Hence, for β slightly larger than β^* , the non-fundamental steady states $(\pm x^*, \omega^*)$ are stable. As β increases, ζ also increases, and $\zeta \rightarrow +\infty$ as $\beta \rightarrow +\infty$. Hence, one of the eigenvalues must cross the unit circle at some critical $\beta = \beta^{**}$ and the non-fundamental steady states E_1 and E_2 become unstable.

Now, for $\beta > \beta^*$,

$$g(1) = \frac{2(1+\delta) + 2\alpha\delta(2-g+v)}{(1+\alpha)}\zeta \quad (59)$$

Which, for $(g+v) < 2 + \left(\frac{1+\delta}{\alpha\delta}\right)$, is positive for all ζ , implying eigenvalues of $\lambda = +1$ are not allowed.

Similarly,

$$g(-1) = -2 - 2\alpha\left(\frac{\delta(g+v)+1}{(1+\alpha)}\right)\zeta \quad (60)$$

is negative for all ζ , so that eigenvalues of $\lambda = -1$ are also not allowed. Thus, as $\zeta \rightarrow \infty$ there must be a complex conjugate pair of eigenvalues crossing the unit circle at $\beta = \beta^{**}$, implying that a Hopf bifurcation occurs at $\beta = \beta^{**}$.

□

Proof of Lemma 3. Under a one off intervention of size $(1+\delta)a\sigma^2\Upsilon_t = \tilde{\theta}$, the steady state deviations from equilibrium, $\pm x^*$, are given by the solutions (if they exist) to

$$\omega^* = \frac{1}{1 + \exp\left[\frac{\beta q}{a\sigma^2}(x^*)^2\left(\delta(g+v) - \frac{\delta(2\alpha+1)+1}{\alpha} - \frac{\tilde{\theta}}{x^*}\right) + \beta C\right]}, \quad (61)$$

in which case the the non-fundamental steady states are gives by

$$x^* = \frac{\mu \pm \sqrt{\mu - 4\vartheta}}{2} \quad (62)$$

with

$$\mu = \tilde{\theta} \left(\frac{\alpha}{1 + \delta(2\alpha + 1) - \alpha\delta(g + v)} \right) \quad (63)$$

$$\vartheta = \frac{a\sigma^2}{\beta q} \left(\frac{\alpha}{1 + \delta(2\alpha + 1) - \alpha\delta(g + v)} \right) \left(\ln \left[\frac{1 - \omega^*}{\omega^*} \right] - \beta C \right). \quad (64)$$

As for the case with no intervention, these values for the non fundamental steady states, x^* , are real if either

$$1. \quad (g + v) < 2 + \left(\frac{1 + \delta}{\alpha\delta} \right) \quad \text{and} \quad \exp(\beta C) > \frac{1 - \omega^*}{\omega^*}$$

or

$$2. \quad (g + v) > 2 + \left(\frac{1 + \delta}{\alpha\delta} \right) \quad \text{and} \quad \exp(\beta C) < \frac{1 - \omega^*}{\omega^*}$$

Furthermore, $-\frac{\mu}{2}$ is the abscissa of the turning point of the polynomial which has x^* as its solution. Clearly this is zero when there is no intervention by the authorities, and the two non-fundamental steady states are equally spaced about zero. When x is positive, the authorities intervene to purchase domestic assets so that θ is negative. This shifts the positive solution of x^* closer to the origin. Similarly, when x is negative the authorities sell domestic assets which shifts the negative solution of x^* closer to the origin.

Stability of the non-fundamental steady states under intervention can be established by examining the characteristic equation

$$g(\lambda) = \lambda^3 - \left(1 + \frac{\alpha(1 - \delta)}{(1 + \alpha)} \zeta \right) \lambda^2 + (1 + \delta) \zeta \lambda - \left(\frac{2\alpha\delta(g + v - 1) - (1 + \delta)}{(1 + \alpha)} - \frac{\alpha(1 + \delta)a\sigma^2\tilde{\theta}}{(1 + \alpha)x^*} \right) \zeta \quad (65)$$

Note firstly that the roots of the general cubic polynomial must satisfy the following conditions:

$$\lambda_1 + \lambda_2 + \lambda_3 = \phi_3 \quad (66)$$

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = -\phi_2 \quad (67)$$

$$\lambda_1\lambda_2\lambda_3 = \phi_1 \quad (68)$$

Furthermore, complex conjugate roots cross the unit circle when

$$\lambda_1\lambda_2 = 1 \quad (69)$$

$$|\lambda_1 + \lambda_2| < 2 \quad (70)$$

Combining these conditions, we find that the following condition must hold when complex conjugate roots cross the unit circle:

$$|\phi_3 - \phi_1| < 2. \quad (71)$$

Substituting in for ϕ_1 and ϕ_3 , this reduces to the condition

$$\left| 1 + \left(\frac{(1 + \delta)(1 + \alpha) - 2\alpha\delta(g + v)}{(1 + \alpha)} \right) \zeta - \frac{\alpha(1 + \delta)a\sigma^2\tilde{\theta}}{(1 + \alpha)x^*} \zeta \right| < 2. \quad (72)$$

For reasonable values of the parameters, the numerator of the first fraction in this condition is positive. Now consider the situation where there is initially no intervention so that $\tilde{\theta} = 0$, and β , and hence ζ is just large for the non-fundamental steady states to become unstable, i.e. $\beta = \beta^{**}$. At the corresponding value of ζ condition (72) will be satisfied. However, whatever the value of ζ we can always find some $\tilde{\theta} = \check{\theta}$ such that (72) no longer holds, and the non-fundamental steady states are stable.

□

B Computation of Size of Effective Interventions

The method for computation of the quantity of intervention necessary to burst a bubble is based on the calculation of nonlinear impulse response functions as described by Koop, Pesaran and Potter (1996).

Impulse response functions trace the effect of a one-off shock on the forecast of variables in a given model. The response of a variable following a shock must

be compared to a baseline 'no shock' scenario. An 'generalized' impulse response function (GRI) of a variable X can be expressed (using Koop et al's notation) as

$$GIR_X(n, v_t, \omega_{t-1}) = E[X_{t+n}|v_t, \omega_{t-1}] - E[X_{t+n}|\omega_{t-1}], \quad n = 0, 1, \dots \quad (73)$$

where n is the forecast horizon, v_t is the shock generating the response, ω_{t-1} is the 'history' or initial values in the model, and $E[\cdot]$ is the expectations operator.

For the purposes of the calculation, an 'effective' intervention is defined as one which reverses the trend of the exchange rate relative to the 'no shock' forecast.

First define the trend indicator variable ι_t as

$$\iota_t = \begin{cases} 1 & \text{if } X_t > 0 \text{ and } E[X_{t+1} - X_t|\omega_{t-1}] > 0 \\ -1 & \text{if } X_t < 0 \text{ and } E[X_{t+1} - X_t|\omega_{t-1}] < 0 \\ 0 & \text{otherwise} \end{cases} \quad (74)$$

so that $\iota_t = 1$ when the baseline forecast of X_t follow a positive bubble path, and $\iota_t = -1$ when the baseline forecast of X_t follows a negative bubble path. Let the level of exchange rate above which a single intervention of magnitude Δ is triggered be denoted by τ . Then, conditional on τ , an intervention at time t of magnitude Δ is identified as 'effective' by the indicator variable \mathcal{I} :

$$\mathcal{I}(\Delta, \tau) = \begin{cases} 1 & \text{if } \iota \times E[X_{t+1} - X_t|\vartheta_t, \omega_{t-1}] < 0 \\ 0 & \text{otherwise.} \end{cases} \quad (75)$$

Thus following an effective intervention an increasing trend in the baseline forecast becomes decreasing in the shocked forecast, while a decreasing trend in the baseline forecast becomes increasing in the shocked forecast.

Due to the nonlinear nature of the model, the response of the variables to shocks must be conditioned on a given initial history. Furthermore, the effects of future shocks to the variables must be drawn from some distribution and averaged out over a large number of draws. In order to deal with these complications, the effects of interventions must be computed by simulating the model, for which the following algorithm is used.

The model is assumed to be known, so that sampling variability is ignored. Baseline forecasts are computed for q periods, and a single shock to the intervention variable ϑ_t of size Δ occurs in period $i < q$ such that $|E[X_{i-1}|\omega_{t-1}]| < \tau < |E[X_i|\omega_{t-1}]|$ and $\iota \neq 0$.

1. Pick an intervention threshold τ .

2. Take an initial history ω_{t-1}^r from the actual values of the variable X at a given date.
3. Take a sequence of shocks v_{t+n}^b , $n = 0, \dots, q$, where the shocks are drawn with replacement from the estimated residuals of the model.
4. Given ω^r and v_{t+n}^b , simulate the evolution of x_{t+n} over $q + 1$ periods. Denote the resulting baseline path $X_{t+n}(\omega_{t-1}^r, v_{t+n}^b)$, $n = 0, \dots, q$.
5. From the baseline forecast, choose a date i such that $|E[X_{i-1}|\omega_{t-1}]| < \tau < |E[X_i|\omega_{t-1}]|$ and $\iota_i \neq 0$. Then set $\vartheta_t = \iota_i \times \Delta^j$ for $t = i$, and $\vartheta_t = 0$ for $t \neq i$. Simulate the evolution of X_{t+n} over $q + 1$ periods and denote the resulting path $X_{t+n}(\vartheta_t, \omega_{t-1}^r, v_{t+n}^b)$, $n = 0, \dots, q$.
6. Calculate

$$\mathcal{I}(\Delta^j, \tau) = \begin{cases} 1 & \text{if } \iota \times E[X_{t+1} - X_t | \vartheta_t, \omega_{t-1}] < 0 \\ 0 & \text{otherwise.} \end{cases}$$

7. Repeat step 5 J times, and denote the smallest value of Δ such that $\mathcal{I}(\Delta^j, \tau) = 1$ by $\Delta_{min}^{b,r}(\tau)$.
8. Repeat steps 3 to 7 B times to obtain the distribution of effective interventions conditional on initial histories from $[\Delta_{min}^{1,r}(\tau), \dots, \Delta_{min}^{B,r}(\tau)]$.
9. Repeat steps 2 to 8 R times, and average the distributions obtained over initial histories to obtain distributions of effective intervention conditional only on the threshold τ

In this paper, in step 1 τ ranged from 0.05 to 0.3 in steps of 0.01. R was set to 30, using the first set of 30 observations from the data, and B was set to 1000. Figure (12) in the main text plots the 25%, 60% and 70% percentiles of the distributions thus obtained against the threshold value τ .